

$$\int (-3x^{-4}) dx = x^{-3} + C \quad (1) \quad (أ)$$

$$\frac{d}{dx}(x^{-3} + C) = -3x^{-4} \quad (ب) \quad \text{تحقق}$$

$$\int (8 + \csc x \cot x) dx = 8x - \csc x + C \quad (2) \quad (أ)$$

$$\frac{d}{dx}(8x - \csc x + C) = 8 + \csc x \cot x \quad (ب) \quad \text{تحقق}$$

$$(3) \int (-x^{-3} + x - 1) dx = \frac{x^{-2}}{2} + \frac{x^2}{2} - x + C$$

$$(4) \int \frac{4}{3} \sqrt[3]{t} dt = \int \frac{4}{3} t^{1/3} dt = t^{4/3} + C$$

$$(5) \int \left(\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}} \right) dx = \int (x^{1/3} + x^{-1/3}) dx = \frac{3}{4} x^{4/3} + \frac{3}{2} x^{2/3} + C$$

$$(6) \int 2 \sec t \tan t dt = 2 \sec t + C$$

$$(7) \int \csc^2 7t dt = -\frac{\cot 7t}{7} + C$$

$$(8) \int \cot^2 t dt = \int (\csc^2 t - 1) dt = -\cot t - t + C$$

$$\frac{dy}{dx} = \frac{1}{x^2} + x \quad (9)$$

$$\int \frac{dy}{dx} dx = \int (x^{-2} + x) dx$$

$$y = -x^{-1} + \frac{x^2}{2} + C$$

$$y = \frac{x^2}{2} - \frac{1}{x} + C$$

$$y(2) = 1 \quad \text{فرضية أساسية}$$

$$1 = \frac{2^2}{2} - \frac{1}{2} + C$$

$$1 = \frac{3}{2} + C$$

$$C = -\frac{1}{2}$$

$$y = \frac{x^2}{2} - \frac{1}{x} - \frac{1}{2} \quad \text{الحل}$$

$$\frac{dy}{dx} = x^{-2/3} \quad (10)$$

$$\int \frac{dy}{dx} dx = \int x^{-2/3} dx$$

$$y = 3x^{1/3} + C$$

فرضية أساسية:

$$y(-1) = -5$$

$$-5 = 3(-1)^{1/3} + C$$

$$-5 = -3 + C$$

$$-2 = C$$

$$y = 3x^{1/3} - 2 \quad \text{الحل:}$$

$$\frac{d^2 y}{d\theta^2} = \sin \theta \quad (11)$$

$$\int \frac{d^2 y}{d\theta^2} d\theta = \int \sin \theta d\theta$$

$$\frac{dy}{d\theta} = -\cos \theta + C_1$$

$$y'(0) = 0$$

$$0 = -\cos 0 + C_1 \quad \text{فرضية أساسية:}$$

$$0 = -1 + C_1$$

$$1 = C_1$$

$$\frac{dy}{d\theta} = -\cos \theta + 1$$

$$\int \frac{dy}{d\theta} d\theta = \int (-\cos \theta + 1) d\theta \quad \text{المشتقة الأولى:}$$

$$y = -\sin \theta + \theta + C_2$$

$$y(0) = -3 \quad \text{فرضية أساسية:}$$

$$-3 = -\sin 0 + 0 + C_2$$

$$-3 = C_2$$

$$y = -\sin \theta + \theta - 3 \quad \text{الحل:}$$

$$\begin{aligned}\frac{d^3 y}{dt^3} &= \frac{1}{t^3} & (12) \\ \int \frac{d^3 y}{dt^3} dt &= \int t^{-3} dt \\ \frac{d^2 y}{dt^2} &= -\frac{1}{2} t^{-2} + C_1\end{aligned}$$

$$y''(1) = 2 \quad \text{فرضية أساسية:}$$

$$2 = -\frac{1}{2} (1)^{-2} + C_1$$

$$2 = -\frac{1}{2} + C_1$$

$$\frac{5}{2} = C_1$$

$$\frac{d^2 y}{dt^2} = -\frac{1}{2} t^{-2} + \frac{5}{2} \quad \text{مشتقة ثانية:}$$

$$\int \frac{d^2 y}{dt^2} dt = \int \left(-\frac{1}{2} t^{-2} + \frac{5}{2} \right) dt$$

$$\frac{dy}{dt} = \frac{1}{2} t^{-1} + \frac{5}{2} t + C_2$$

$$y'(1) = 3 \quad \text{فرضية أساسية:}$$

$$3 = \frac{1}{2} (1)^{-1} + \frac{5}{2} (1) + C_2$$

$$3 = 3 + C_2$$

$$0 = C_2$$

$$\int \frac{dy}{dx} = \frac{1}{2} t^{-1} + \frac{5}{2} t \quad \text{مشتقة أولى:}$$

$$\int \frac{dy}{dt} dt = \int \left(\frac{1}{2} t^{-1} + \frac{5}{2} t \right) dt$$

$$y = \frac{1}{2} \ln |t| + \frac{5}{4} t^2 + C_3$$

$$y(1) = 1 \quad \text{فرضية أساسية:}$$

$$1 = \frac{1}{2} \ln 1 + \frac{5}{4} (1)^2 + C_3$$

$$1 = \frac{5}{4} + C_3$$

$$-\frac{1}{4} = C_3$$

$$y = \frac{1}{2} \ln |t| + \frac{5}{4} t^2 - \frac{1}{4} \quad \text{الحل:}$$

$$\frac{dv}{dt} = a = 32 \quad (13)$$

$$\int \frac{dv}{dt} dt = \int 32 dt$$

$$v = 32t + C_1$$

$$v(0) = 20 \quad \text{فرضية أساسية:}$$

$$20 = 32(0) + C_1$$

$$20 = C_1$$

$$\frac{ds}{dt} = v = 32t + 20 \quad \text{السرعة اللحظية:}$$

$$\int \frac{ds}{dt} dt = \int (32t + 20) dt$$

$$s = 16t^2 + 20t + C_2$$

$$s(0) = 0 \quad \text{فرضية أساسية:}$$

$$0 = 16(0)^2 + 20(0) + C_2$$

$$C_2 = 0$$

$$s = 16t^2 + 20t \quad \text{الحل:}$$

$$\frac{dy}{dx} = x - \frac{1}{x^2} \quad (14) \quad (i)$$

$$\int \frac{dy}{dx} dx = \int (x - x^{-2}) dx$$

$$y = \frac{x^2}{2} + x^{-1} + C = \frac{x^2}{2} + \frac{1}{x} + C$$

$$y(1) = 2 \quad \text{فرضية أساسية:}$$

$$2 = \frac{1^2}{2} + \frac{1}{1} + C$$

$$2 = \frac{3}{2} + C$$

$$\frac{1}{2} = C$$

$$y = \frac{x^2}{2} + \frac{1}{x} + \frac{1}{2}, \quad \text{الحل: } x > 0$$

$$y = \frac{x^2}{2} + \frac{1}{x} + C \quad \text{(ب) مجددًا،}$$

$$y(-1) = 1 \quad \text{فرضية أساسية:}$$

$$1 = \frac{(-1)^2}{2} + \frac{1}{(-1)} + C$$

$$1 = -\frac{1}{2} + C$$

$$\frac{3}{2} = C$$

$$y = \frac{x^2}{2} + \frac{1}{x} + \frac{3}{2}, \quad \text{الحل: } x < 0$$

$$x < 0, \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{x} + \frac{x^2}{2} + C_1\right) = -\frac{1}{x^2} + x = x - \frac{1}{x^2} \quad \text{ج) ل}$$

$$x > 0, \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{x} + \frac{x^2}{2} + C_2\right) = -\frac{1}{x^2} + x = x - \frac{1}{x^2} \quad \text{ل}$$

$$\frac{dy}{dx}, x = 0 \text{ غير معرفة.}$$

(د) لتكن C_1 قيمة من الجزء (ب) و C_2 قيمة من الجزء (ا). لذلك، $C_1 = \frac{3}{2}$ و $C_2 = \frac{1}{2}$

$$y(2) = -1$$

$$-1 = \frac{1}{2} + \frac{2^2}{2} + C_2$$

$$-1 = \frac{5}{2} + C_2$$

$$-\frac{7}{2} = C_2$$

$$y(-2) = 2$$

$$2 = \frac{1}{(-2)} + \frac{(-2)^2}{2} + C_1$$

$$2 = \frac{3}{2} + C_1$$

$$\frac{1}{2} = C_1$$

$$\text{لذلك، } C_1 = \frac{1}{2} \text{ و } C_2 = -\frac{7}{2}$$

$$\int \frac{dc}{dx} dx = \int (3x^2 - 12x + 15) dx \quad (15)$$

$$c = x^3 - 6x^2 + 15x + C$$

$$c(0) = 400$$

فرضية أساسية:

$$400 = 0^3 - 6(0)^2 + 15(0) + C$$

$$400 = C$$

$$\text{الحل: } c(x) = x^3 - 6x^2 + 15x + 400$$

$$) dx = \int \frac{d}{dx} (x \sin x) dx = x \sin x + C$$

$$(b) \int (-g(x)) dx = - \int g(x) dx = -x \sin x + C$$

$$(c) \int (f(x) - g(x)) dx = \int f(x) dx - \int g(x) dx = x^2 - x \sin x + C$$

$$(d) \int (g(x) - 4) dx = \int g(x) dx - \int 4 dx = x \sin x - 4x + C$$