1. $u = 3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$\int \sin 3x\,dx = \frac{1}{3}\int \sin u\,du = -\frac{1}{3}\cos u + C = -\frac{1}{3}\cos 3x + C$

تحقق:

$\frac{d}{dx}(-\frac{1}{3}\cos 3x + C) = -\frac{1}{3}(-\sin 3x)(3) = \sin 3x$

2. $u = 2x$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$\int \sec 2x \tan 2x\,dx = \frac{1}{2}\int \sec u \tan u\,du = \frac{1}{2}\sec u + C = \frac{1}{2}\sec 2x + C$

تحقق:

$\frac{d}{dx}(\frac{1}{2}\sec 2x + C) = \frac{1}{2}\sec 2x \tan 2x^2 = \sec 2x \tan 2x$

3. $u = 2x + 3 \Rightarrow du = 2dx$

$dx = \frac{1}{2}du$

$\int \frac{1}{2}u^{-\frac{1}{2}}du = \frac{1}{2}\frac{u^{-\frac{1}{2}}}{\frac{1}{2}} + C = \sqrt{2x+3} + C$

تحقق:

$\frac{d}{dx}(\sqrt{2x+3} + c) = \frac{2}{2\sqrt{2x+3}} = \frac{1}{\sqrt{2x+3}}$

4. $u = y^4 + 4y^2 + 1$

$du = (4y^3 + 8y)\,dy$

$du = 4(y^3 + 2y)\,dy$

$\frac{1}{4}du = (y^3 + 2y)\,dy$

$\int 8(y^4 + 4y^2 + 1)^2(y^3 + 2y)\,dy = 8(\frac{1}{4})\int u^2\,du = \frac{2}{3}u^3 + C = \frac{2}{3}(y^4 + 4y^2 + 1)^3 + C$

تحقق:

$\frac{d}{dx}[\frac{2}{3}(y^4 + 4y^2 + 1)^3 + C] = 2(y^4 + 4y^2 + 1)^2(4y^3 + 8y) = 8(y^4 + 4y^2 + 1)^2(y^3 + 2y)$

5. $u = 1 - x$

$du = - dx$

$$\int \frac{dx}{(1-x)^2} = -\int \frac{du}{u^2} = u^{-1} + C = \frac{1}{1-x} + C$$

6. $u = \ln x\,; du = \dfrac{dx}{x}$

$x = e\,; u = 1$

$x = 6\,; u = \ln 6$

$$\int_1^{\ln 6} \frac{du}{u} = \ln|u|\Big]_1^{\ln 6} = \ln(\ln 6)$$

7. $u = 3z + 4;\ du = 3z$

$dz = \dfrac{1}{3} du$

$$\int \frac{1}{3}\cos u\, du = \frac{1}{3}\sin u + C = \frac{1}{3}\sin(3z+4) + C$$

8. $u = \ln x$

$du = \dfrac{1}{x}\, dx$

$$\int \frac{\ln^6 x}{x}\, dx = \int u^6\, du = \frac{1}{7} u^7 + C = \frac{1}{7}(\ln^7 x) + C$$

9. $u = s^{4/3} - 8$

$du = \dfrac{4}{3} s^{1/3}\, ds$

$\dfrac{3}{4} du = s^{1/3}\, ds$

$$\int s^{1/3} \cos(s^{4/3} - 8)\, ds = \frac{3}{4}\int \cos u\, du = \frac{3}{4}\sin u + C = \frac{3}{4}\sin(s^{4/3} - 8) + C$$

10. $\displaystyle\int_{\pi/4}^{3\pi/4} \cot x\, dx = \int_{\pi/4}^{3\pi/4} \frac{\cos x}{\sin x}\, dx$

$u = \sin x$

$du = \cos x\, dx$

$$\int_{\pi/4}^{3\pi/4} \frac{\cos x}{\sin x}\, dx = \int_{x=\pi/4}^{x=3\pi/4} \frac{1}{u}\, du = \ln|u|\Big]_{x=\pi/4}^{x=3\pi/4} = \ln|\sin x|\Big]_{\pi/4}^{3\pi/4} = \ln\left|\frac{\sqrt{2}}{2}\right| - \ln\left|\frac{\sqrt{2}}{2}\right| = 0$$

11. $u = x^2 + 1$

$du = 2x\, dx$

$x\, dx = \dfrac{1}{2}\, du$

$$\int_{-1}^{3} \frac{x\, dx}{x^2+1} = \frac{1}{2}\int_2^{10} \frac{1}{u}\, du = \frac{1}{2}\ln|u|\Big]_2^{10} = \frac{1}{2}(\ln 10 - \ln 2) = \frac{1}{2}\ln 5 \approx 0.805$$

12. $u = 5x + 8$

$du = 5\,dx$

$\frac{1}{5}\,du = dx$

$$\int \frac{dx}{\sqrt{5x+8}} = \frac{1}{5}\int u^{-1/2}\,du = \frac{1}{5}\cdot 2u^{1/2} + C = \frac{2}{5}\sqrt{5x+8+C}$$

13. $u = y + 1$

$du = dy$

$$\int_0^3 \sqrt{y+1}\,dy = \int_1^4 u^{1/2}\,du = \frac{2}{3}u^{3/2}\Big]_1^4 = \frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2} = \frac{2}{3}(8) - \frac{2}{3} = \frac{14}{3}$$

14. $u = 1 + \theta^{3/2}$

$du = \frac{3}{2}\theta^{1/2}\,d\theta$

$\frac{2}{3}\,du = \theta^{\frac{1}{2}}\,d\theta$

$$\int_0^1 \frac{10\sqrt{\theta}}{(1+\theta^{3/2})^2}\,d\theta = \frac{2}{3}(10)\int_1^2 u^{-2}\,du = -\frac{20}{3}u^{-1}\Big]_1^2 = -\frac{20}{3}\left(\frac{1}{2} - 1\right) = -\frac{20}{3}\left(-\frac{1}{2}\right) = \frac{10}{3}$$

15. $u = t^5 + 2t$

$du = (5t^4 + 2)\,dt$

$$\int_0^1 \sqrt{t^5+2t}\,(5t^4+2)\,dt = \int_0^3 u^{1/2}\,du = \frac{2}{3}u^{3/2}\Big]_0^3 = \frac{2}{3}(3)^{3/2} = \frac{2}{3}\sqrt{27} = 2\sqrt{3}$$

16. $\frac{dy}{dx} = (y+5)(x+2)$

$\frac{dy}{y+5} = (x+2)\,dx$

كامل كلتا الجهتين: $\int \frac{dy}{y+5} = \int (x+2)\,dx$

$\ln|y+5| = \frac{1}{2}x^2 + 2x + C$

$|y+5| = e^{\frac{1}{2}x^2 + 2x + C}$

$$|y+5| = e^c e^{(1/2)x^2+2x}$$

لتكن الآن، $C = e^c$ أو $C = -e^c$، إذا $(y+5)$ موجبة أو سالبة.

ثم، $y + 5 = Ce^{\frac{1}{2}x^2+2x}$ ومنها $y = C\,e^{(1/2)x^2+2x} - 5$

17. $\frac{dy}{dx} = (\cos x)e^{y+\sin x}$

$$\frac{dy}{dx} = (\cos x)(e^y e^{\sin x})$$

$$\frac{dy}{e^y} = \cos x\, e^{\sin x}\, dx$$

كامل كلتا الجهتين:

$$\int \frac{dy}{e^y} = \int \cos x\, e^{\sin x}\, dx$$

إلى جهة اليمين،

$$u = \sin x$$

$$du = \cos x\, dx$$

$$-e^{-y} = \int e^u\, du$$

$$-e^{-y} = e^u + C$$

$$-e^{-y} = e^{\sin x} + C$$

$$e^{-y} = -e^{\sin x} + C$$

$$-y = \ln(C - e^{\sin x})$$

$$y = -\ln(C - e^{\sin x})$$

18. $\frac{dy}{dx} = -2xy^2$

$$-\frac{dy}{y^2} = 2x\, dx$$

$$-\int \frac{dy}{y^2} = \int 2x\, dx$$

$$y^{-1} = x^2 + C$$

$$y = \frac{1}{x^2+C}$$

$$y(1) = \frac{1}{1+C} = 0.25$$

$$1 + C = 4$$

$$C = 3;\ y = \frac{1}{x^2+3}$$

19 $u = 1 - \cos 3x$ ، $du = 3\sin 3x\, dx$

(أ) $\int_{\pi/6}^{\pi/3} (1 - \cos 3x) \sin 3x\, dx = \int_1^2 \frac{1}{3} u\, du = \frac{1}{6} u^2 \Big]_1^2 = \frac{1}{6}(2)^2 - \frac{1}{6}(1)^2 = \frac{1}{2}$

(ب) $\int (1 - \cos 3x) \sin 3x\, dx = \int \frac{1}{3} u\, du = \frac{1}{6}u^2 + C = \frac{1}{6}(1 - \cos 3x)^2 + C$

$$\int_{\pi/6}^{\pi/3} (1 - \cos 3x) \sin 3x\, dx = \frac{1}{6}(1 - \cos 3x)^2 \Big]_{\pi/6}^{\pi/3} = \frac{1}{6}(2)^2 - \frac{1}{6}(1)^2 = \frac{1}{2}$$

.20 $u = x^4 + 9$ ، $du = 4x^3 dx$

(أ) $\int_0^1 \frac{x^3\, dx}{\sqrt{x^4 + 9}} = \int_9^{10} \frac{1}{4} u^{-1/2}\, du = \frac{1}{2} u^{1/2} \Big]_9^{10} = \frac{1}{2}\sqrt{10} - \frac{1}{2}\sqrt{9} = \frac{1}{2}\sqrt{10} - \frac{3}{2} \approx 0.081$

(ب) $\int \frac{x^3}{x^4 + 9} dx = \int \frac{1}{4} u^{-1/2}\, du = \frac{1}{2} u^{1/2} + C = \frac{1}{2}\sqrt{x^4 + 9} + C$

$$\int_0^1 \frac{x^3}{x^4 + 9} dx = \frac{1}{2}\sqrt{x^4 + 9}\Big]_0^1 = \frac{1}{2}\sqrt{10} - \frac{1}{2}\sqrt{9} = \frac{1}{2}\sqrt{10} - \frac{3}{2} \approx 0.081$$