1. $u = 2x^2$

$du = 4x\,dx$

$x\,dx = \frac{1}{4}\,du$

$\int x\cos(2x^2)\,dx = \frac{1}{4}\int \cos u\,du = \frac{1}{4}\sin u + C = \frac{1}{4}\sin(2x^2) + C$

تحقق:

$\frac{d}{dx}(\frac{1}{4}\sin(2x^2) + C) = \frac{1}{4}\cos(2x^2)(4x) = x\cos(2x^2)$

2. $u = 7x - 2$

$du = 7\,dx$

$\frac{1}{7}\,du = dx$

$\int 28(7x-2)^3\,dx = \frac{1}{7}\int 28u^3\,du = u^4 + C = (7x-2)^4 + C$

تحقق:

$\frac{d}{dx}[(7x-2)^4 + C] = 4(7x-2)^3(7) = 28(7x-2)^3$

3. $u = 1 - r^3$

$du = -3r^2\,dr$

$-\frac{1}{3}\,du = r^2\,dr$

$\int \frac{9r^2\,dr}{\sqrt{1-r^3}} = 9(-\frac{1}{3})\int \frac{du}{\sqrt{u}} = -3\int u^{-1/2}\,du = -3(2)u^{1/2} + C = -6\sqrt{1-r^3} + C$

تحقق:

$\frac{d}{dx}(-6\sqrt{1-r^3} + C) = -6(\frac{1}{2\sqrt{1-r^3}})(-3r^2) = \frac{9r^2}{\sqrt{1-r^3}}$

4. $u = 1 - \cos\frac{t}{2}$

$du = \frac{1}{2}\sin\frac{t}{2}\,dt$

$2\,du = \sin\frac{t}{2}\,dt$

$\int (1-\cos\frac{t}{2})^2\sin\frac{t}{2}\,dt = 2\int u^2\,du = \frac{2}{3}u^3 + C = \frac{2}{3}(1-\cos\frac{t}{2})^3 + C$

تحقق:

$\frac{d}{dx}[\frac{2}{3}(1-\cos\frac{t}{2})^3 + C] = 2(1-\cos\frac{t}{2})^2(\sin\frac{t}{2})(\frac{1}{2}) = (1-\cos\frac{t}{2})^2\sin\frac{t}{2}$

5. $u = x + 2$

$du = dx$

$\int \sec^2(x+2)\,dx = \int \sec^2 u\,du = \tan u + C = \tan(x+2) + C$

6. $\int \frac{dx}{\sin^2 3x} = \int \csc^2 3x\,dx$

$u = 3x$

$du = 3dx$

$\frac{1}{3}du = dx$

$\int \csc^2 3x\,dx = \frac{1}{3}\int \csc^2 u\,du = -\frac{1}{3}\cot u + C = -\frac{1}{3}\cot(3x) + C$

7. $u = x + 2$

$du = dx$

$\int_0^7 \frac{dx}{x+2} = \int_2^9 \frac{1}{u}\,du = \ln|u|\Big]_2^9 = \ln 9 - \ln 2 \approx 1.504$

8. $\int \frac{dx}{\cot 3x} = \int \frac{\sin 3x}{\cos 3x}\,dx$

$u = \cos 3x$

$du = -3\sin 3x\,dx$

$-\frac{1}{3}du = \sin 3x\,dx$

$\int \frac{dx}{\cot 3x} = -\frac{1}{3}\int \frac{1}{u}\,du = -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|\cos 3x| + C$

(المعادلة المكافئة هي: $\frac{1}{3}\ln|\sec 3x| + C$)

9. $u = 1 - r^2$

$du = -2r\,dr$

$-\frac{1}{2}\,du = r\,dr$

$\int_0^1 r\sqrt{1-r^2}\,dr = -\frac{1}{2}\int_1^0 u^{1/2}\,du = -\frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big]_1^0 = -\frac{1}{3}(0) + \frac{1}{3}(1) = \frac{1}{3}$

10. $u = 4 + r^2$

$du = 2r\,dr$

$\frac{1}{2}\,du = r\,dr$

$\int_{-1}^1 \frac{5r}{(4+r^2)^2}\,dr = \frac{5}{2}\int_5^5 u^{-2}\,du = 0$

الوحدة الرابعة

11. $\dfrac{dy}{dx} = x\sqrt{y}\cos^2\sqrt{y}$

$$\frac{dy}{\sqrt{y}\cos^2\sqrt{y}} = x\,dx$$

كامل كلتا الجهتين:

$$\int \frac{dy}{\sqrt{y}\cos^2\sqrt{y}} = \int x\,dx$$

إلى جهة اليسار،

$$u = \sqrt{y}$$

$$du = \frac{1}{2}y^{-1/2}\,dy$$

$$2\,du = y^{-1/2}\,dy$$

$$2\int \frac{du}{\cos^2 u} = \frac{1}{2}x^2 + C$$

$$2\int \sec^2 u\,du = \frac{1}{2}x^2 + C$$

$$2\tan u = \frac{1}{2}x^2 + C$$

$$2\tan\sqrt{y} = \frac{1}{2}x^2 + C$$

$$\tan\sqrt{y} = \frac{1}{4}x^2 + C$$

12. $\dfrac{dy}{dx} = e^{x-y}$

$$\frac{dy}{dx} = e^x e^{-y}$$

$$\frac{dy}{e^{-y}} = e^x\,dx$$

كامل كلتا الجهتين

$$\int \frac{dy}{e^{-y}} = \int e^x\,dx$$

$$\int e^y\,dy = \int e^x\,dx$$

$$e^y = e^x + C$$

$$y = \ln(e^x + C)$$

13. $\dfrac{dy}{dx} = \dfrac{4\sqrt{y}\ln x}{x}$

$$\frac{dy}{\sqrt{y}} = 4\frac{\ln x}{x}\,dx$$

كامل كلتا الجهتين $\displaystyle\int \frac{dy}{\sqrt{y}} = 4\int \frac{\ln x}{x}\,dx$

إلى جهة اليمين، لتكن $u = \ln x$

$$du = \frac{1}{x}\,dx$$
$$2y^{1/2} = 4\int u\,du$$
$$2y^{1/2} = 4(\frac{1}{2}u^2) + C$$
$$2y^{1/2} = 2(\ln x)^2 + C$$
$$y^{1/2} = (\ln x)^2 + C$$
$$y = [(\ln x)^2 + C]^2$$
$$y(e) = [(\ln e)^2 + C]^2 = 1$$
$$(1 + C)^2 = 1$$
$$C = 0$$
$$y = (\ln x)^4$$

إرشاد: علامات القيمة المطلقة غير ضرورية لأن المسألة الأصلية تتعلق بـ $\ln x$ إذًا نعلم أن $x > 0$.

14. نبين أن $f'(x) = \tan x$ و $f(3) = 5$، حيث

$$f(x) = \left(\ln\left|\frac{\cos 3}{\cos x}\right| + 5\right)$$

$$f'(x) = \frac{d}{dx}\left(\ln\left|\frac{\cos 3}{\cos x}\right| + 5\right) = \frac{\mathrm{d}}{\mathrm{dx}}(\ln|\cos 3| - \ln|\cos x| + 5) = -\frac{1}{\cos x}(-\sin x) = \tan x$$

$$f(3) = \ln\left|\frac{\cos 3}{\cos 3}\right| + 5 = \ln 1 + 5 = 5$$