1. $dv = \sin x\,dx$

$v = -\cos x$

$u = x$

$du = dx$

$$\int x \sin x\,dx = -x\cos x + \int \cos x\,dx = -x\cos x + \sin x + C$$

تحقق:

$$\frac{d}{dx}(-x\cos x + \sin x + C) = (-x)(-\sin x) + (\cos x)(-1) + \cos x = x\sin x$$

2. $dv = y\,dy$

$v = \frac{1}{2}y^2$

$u = \ln y$

$du = \frac{1}{y}\,dy$

$$\int y\ln y\,dy = \frac{1}{2}y^2\ln y - \int \frac{1}{2}y^2 \cdot \frac{1}{y}\,dy = \frac{1}{2}y^2\ln y - \frac{1}{2}\int y\,dy = \frac{1}{2}y^2\ln y - \frac{1}{4}y^2 + C$$

تحقق:

$$\frac{d}{dy}\left[\frac{1}{2}y^2\ln y - \frac{1}{4}y^2 + C\right] = \left(\frac{1}{2}y^2\right)\left(\frac{1}{y}\right) + (\ln y)(y) - \frac{1}{2}y = y\ln y$$

3. $dv = \sec^2 x\,dx$

$v = \tan x$

$u = x$

$du = dx$

$$\int x\sec^2 x\,dx = x\tan x - \int \tan x\,dx = x\tan x - \int \frac{\sin x}{\cos x}\,dx = x\tan x + \ln|\cos x| + C$$

4. $dv = x^3\,dx$

$v = \frac{1}{4}x^4$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\int x^3\ln x\,dx = \frac{1}{4}x^4\ln x - \frac{1}{4}\int x^4\left(\frac{1}{x}\right)dx = \frac{1}{4}x^4\ln x - \frac{1}{4}\int x^3\,dx = \frac{1}{4}x^4\ln x - \frac{1}{16}x^4 + C$$

5. $dv = e^x\,dx$ لتكن: $u = x^2 - 5x$

$v = e^x$ $du = (2x - 5)\,dx$

$$\int (x^2 - 5x)e^x\,dx = (x^2 - 5x)e^x - \int e^x(2x - 5)\,dx$$

$dv = e^x\,dx$

$v = e^x$

$u = 2x - 5$

$du = 2\,dx$

$$(x^2 - 5x)e^x - \int e^x(2x - 5)\,dx = (x^2 - 5x)e^x - (2x - 5)e^x + \int 2e^x\,dx$$

$$= (x^2 - 5x)e^x - (2x - 5)e^x + 2e^x + C = (x^2 - 7x + 7)e^x + C$$

6. $dv = \sin y\,dy$

$v = -\cos y$

$u = e^y$

$du = e^y\,dy$

$$\int e^y \sin y\,dy = -e^y \cos y + \int \cos y\, e^y\,dy$$

$u = e^y$

$du = e^y\,dy$

$dv = \cos y\,dy\,;\ v = \sin y$

$$\int e^y \sin y\,dy = -e^y \cos y + e^y \sin y - \int \sin y\, e^y\,dy$$

$$2\int e^y \sin y\,dy = -e^y \cos y + e^y \sin y$$

$$\int e^y \sin y\,dy = \frac{1}{2} e^y(\sin y - \cos y) + C$$

7. استخدم التكامل الجدولي مع $f(x) = x^2$ و $g(x) = \sin 2x$

$f(x)$ والمشتقّات		$g(x)$ والتكامل
f x^2	+	$\sin 2x$
f' $2x$	−	$-\frac{1}{2}\cos 2x$
f" 2	+	$-\frac{1}{4}\sin 2x$
f''' 0		$\frac{1}{8}\cos 2x$

$$\int x^2 \sin 2x\,dx = -\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + C = \left(\frac{1 - 2x^2}{4}\right)\cos 2x + \frac{x}{2}\sin 2x + C$$

$$\int_0^{\pi/2} x^2 \sin 2x\,dx = \left[\left(\frac{1 - 2x^2}{4}\right)\cos 2x + \frac{x}{2}\sin 2x\right]_0^{\pi/2}$$

$$= \left(\frac{1 - 2\left(\frac{\pi}{2}\right)^2}{4}\right)(-1) + 0 - \frac{1}{4}(1) - 0 = \frac{\pi^2}{8} - \frac{1}{2} \approx 0.734$$

8. $dv = \cos 3x\, dx$

$v = \frac{1}{3}\sin 3x$

$u = e^{2x}$

$du = 2e^{2x}\, dx$

$$\int e^{2x}\cos 3x\, dx = \left(e^{2x}\right)\left(\frac{1}{3}\sin 3x\right) - \int\left(\frac{1}{3}\sin 3x\right)(2e^{2x}\, dx) = \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\int e^{2x}\sin 3x\, dx$$

$dv = \sin 3x\, dx$

$v = -\frac{1}{3}\cos 3x$

$u = e^{2x}$

$du = 2e^{2x}\, dx$

$$\int e^{2x}\cos 3x\, dx = \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\left[\left(e^{2x}\right)\left(-\frac{1}{3}\cos 3x\right) - \int\left(-\frac{1}{3}\cos 3x\right)(2e^{2x}\, dx)\right]$$

$$= \frac{1}{9}e^{2x}(3\sin 3x + 2\cos 3x) - \frac{4}{9}\int e^{2x}\cos 3x\, dx$$

$$\frac{13}{9}\int e^{2x}\cos 3x\, dx = \frac{1}{9}e^{2x}(3\sin 3x + 2\cos 3x)$$

$$\int e^{2x}\cos 3x\, dx = \frac{1}{13}e^{2x}(3\sin 3x + 2\cos 3x)$$

$$\int_{-2}^{3} e^{2x}\cos 3x\, dx = \left[\frac{1}{13}e^{2x}(3\sin 3x + 2\cos 3x)\right]_{-2}^{3}$$

$$= \frac{1}{13}\left[e^{6}(3\sin 9 + 2\cos 9) - e^{-4}(3\sin(-6) + 2\cos(-6)\right]$$

$$= \frac{1}{13}\left[e^{6}(2\cos 9 + 3\sin 9) - e^{-4}(2\cos 6 - 3\sin 6)\right] \approx -18.186$$

9. $y = \int x^2 e^{4x}\, dx$

$dv = e^{4x}\, dx$

$v = \frac{1}{4}e^{4x}$

$u = x^2$

$du = 2x\, dx$

$$y = (x^2)\left(\frac{1}{4}e^{4x}\right) - \int\left(\frac{1}{4}e^{4x}\right)(2x\, dx) = \frac{1}{4}x^2 e^{4x} - \frac{1}{2}\int xe^{4x}\, dx$$

$dv = e^{4x}\, dx$

$$v = \frac{1}{4}e^{4x}$$

$$u = x$$

$$du = dx$$

$$y = \frac{1}{4}x^2e^{4x} - \frac{1}{2}\left[(x)\left(\frac{1}{4}e^{4x}\right) - \int\left(\frac{1}{4}e^{4x}\right)dx\right]$$

$$y = \frac{1}{4}x^2e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{32}e^{4x} + C$$

$$y = \left(\frac{x^2}{4} - \frac{x}{8} + \frac{1}{32}\right)e^{4x} + C$$

10. $y = \int \frac{1}{\cos\theta} \times \frac{\sin\theta}{\cos\theta}d\theta = -\int u^{-2}du = \frac{1}{u} + C = \frac{1}{\cos\theta} + C$, (حيث $u = \cos\theta$)

11. $dv = \sin x\, dx$

$v = -\cos x$

$u = x$

$du = dx$

$$\int x\sin x\, dx = -x\cos x + \int \cos x\, dx = -x\cos x + \sin x + C$$

(a) $\int_0^{\pi} |x\sin x|dx = \int_0^{\pi} x\sin x\, dx = [-x\cos x + \sin x]_0^{\pi} = -\pi(-1) + 0 + 0(1) - 0 = \pi$

(b) $\int_{\pi}^{2\pi} |x\sin x|dx = -\int_{\pi}^{2\pi} x\sin x\, dx = [x\cos x - \sin x]_{\pi}^{2\pi} = 2\pi(1) - 0 - \pi(-1) + 0 = 3\pi$

(c) $\int_0^{2\pi} |x\sin x|dx = \int_0^{\pi} |x\sin x|\, dx + \int_{\pi}^{2\pi} |x\sin x|dx = \pi + 3\pi = 4\pi$

12. لتكن $w = \sqrt{x}$ ومنها $dw = \frac{dx}{2\sqrt{x}}$، إذًا $dx = 2\sqrt{x}\, dw = 2w\, dw$.

$$\int \sin\sqrt{x}\, dx = \int (\sin w)(2w\, dw) = 2\int w\sin w\, dw$$

$dv = \sin w\, dw$

$v = -\cos w$

$u = w$

$du = dw$

$$\int w\sin w\, dw = -w\cos w + \int \cos w\, dw = -w\cos w + \sin w + C$$

$$\int \sin\sqrt{x}\, dx = 2\int w\sin w\, dw = -2w\cos w + 2\sin w + C = -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$$