1. $dv = \cos x\, dx$

$v = \sin x$

$u = x^2$

$du = 2x\, dx$

$\int x^2 \cos x\, dx = x^2 \sin x - 2\int x \sin x\, dx$

$u = x$

$du = dx$

$dv = \sin x dx$

$v = -\cos x$

$= x^2 \sin x - 2[-x\cos x + \sin x] + C = 2x\cos x + (x^2 - 2)\sin x + C$

تحقق:

$$\frac{d}{dx}\left[2x\cos x + (x^2-2)\sin x + C\right] = (2x)(-\sin x) + (2\cos x)(1) + (x^2-2)(\cos x) + (\sin x)(2x)$$
$$= x^2\cos x$$

2. $dv = \sin t\, dt$

$v = -\cos t$

$u = t^2$

$du = 2t\, dt$

$\int t^2 \sin t\, dt = -t^2\cos t + 2\int (\cos t)(t)\, dt$

$dv = \cos t\, dt$

$v = \sin t$

$u = t$

$du = dt$

$$-t^2\cos t + 2\int t\cos t\, dt = -t^2\cos t + 2t\sin t - 2\int \sin t\, dt = -t^2\cos t + 2t\sin t + 2\cos t + C$$
$$= \left(2 - t^2\right)\cos t + 2t\sin t + C$$

3. $dv = \csc^2 t\, dt$

$v = -\cot t$

$u = t$

$du = dt$

$$\int t\csc^2 t\, dt = -t\cot t + \int \cot t\, dt = -t\cot t + \int \frac{\cos t}{\sin t}\, dt = -t\cot t + \ln|\sin t| + C$$

4. استخدم التكامل الجدولي مع $f(x) = x^4$ و $g(x) = e^{-x}$

$$\int x^4 e^{-x}\, dx = -x^4 e^{-x} - 4x^3 e^{-x} - 12x^2 e^{-x} - 24xe^{-x} - 24e^{-x} + C$$
$$= -(x^4 + 4x^3 + 12x^2 + 24x + 24)e^{-x} + C$$

5. استخدم التكامل الجدولي مع $f(x) = x^3$ و $g(x) = e^{-2x}$

$$\int x^3 e^{-2x}\, dx = -\frac{1}{2}x^3 e^{-2x} - \frac{3}{4}x^2 e^{-2x} - \frac{3}{4}x\, e^{-2x} - \frac{3}{8}\, e^{-2x} + C = -\left(\frac{x^3}{2} + \frac{3x^2}{4} + \frac{3x}{4} + \frac{3}{8}\right)e^{-2x} + C$$

6. $dv = \cos y\, dy$

$v = \sin y$

$u = e^{-y}$

$du = -e^{-y}\, dy$

$\int e^{-y} \cos y\, dy = e^{-y} \sin + \int \sin y\, e^{-y}\, dy$

$dv = \sin y\, dy$

$v = -\cos y$

$u = e^{-y}$

$du = -e^{-y}\, dy$

$\int e^{-y} \cos y\, dy = e^{-y} \sin y - e^{-y} \cos y - \int e^{-y} \cos y\, dy$

$2\int e^{-y} \cos y\, dy = e^{-y} \sin y - e^{-y} \cos y$

$\int e^{-y} \cos y\, dy = \frac{1}{2}e^{-y}(\sin y - \cos y) + C$

7. استخدم التكامل الجدولي مع $f(x) = x^3$ و $g(x) = \cos 2x$

$$\int x^3 \cos 2x\, dx = \frac{1}{2}x^3 \sin 2x + \frac{3}{4}x^2 \cos 2x - \frac{3}{4}x \sin 2x - \frac{3}{8} \cos 2x$$
$$= \left(\frac{x^3}{2} - \frac{3x}{4}\right)\sin 2x + \left(\frac{3x^2}{4} - \frac{3}{8}\right)\cos 2x + C$$

$$\int_0^{\pi/2} x^3 \cos 2x\, dx = \left[\left(\frac{x^3}{2} - \frac{3x}{4}\right)\sin 2x + \left(\frac{3x^2}{4} - \frac{3}{8}\right)\cos 2x\right]_0^{\pi/2}$$
$$= 0 + \left(\frac{3\pi^2}{16} - \frac{3}{8}\right)(-1) - 0 - \left(-\frac{3}{8}\right)(1) = \frac{3}{4} - \frac{3\pi^2}{16} \approx -1.101$$

الوحدة الرابعة

8. $dv = \sin 2x\, dx$

$v = \frac{1}{2}\cos 2x$

$u = e^{-2x}$

$du = -2e^{2x}\, dx$

$$\int e^{-2x}\sin 2x\, dx = (e^{-2x})\left(-\frac{1}{2}\cos 2x\right) - \int\left(-\frac{1}{2}\cos 2x\right)(-2e^{-2x}\, dx)$$

$$= -\frac{1}{2}e^{-2x}\cos 2x - \int e^{-2x}\cos 2x\, dx$$

$dv = \cos 2x\, dx$

$v = \frac{1}{2}\sin 2x$

$u = e^{-2x}$

$du = -2e^{-2x}$

$$\int e^{-2x}\sin 2x\, dx = -\frac{1}{2}e^{-2x}\cos 2x - \left[(e^{-2x})\left(\frac{1}{2}\sin 2x\right) - \int\left(\frac{1}{2}\sin 2x\right)(-2e^{-2x}\, dx)\right]$$

$$= -\frac{1}{2}e^{-2x}(\cos 2x + \sin 2x) - \int e^{-2x}\sin 2x\, dx$$

$$2\int e^{-2x}\sin 2x\, dx = -\frac{1}{2}e^{-2x}(\cos 2x + \sin 2x) + C$$

$$\int e^{-2x}\sin 2x\, dx = -\frac{e^{-2x}}{4}(\cos 2x + \sin 2x) + C$$

$$\int_{-3}^{2} e^{-2x}\sin 2x dx = \left[-\frac{e^{-2x}}{4}(\cos 2x + \sin 2x)\right]_{-3}^{2}$$

$$= -\frac{e^{-4}}{4}(\cos 4 + \sin 4) + \frac{e^{6}}{4}[\cos(-6) + \sin(-6)]$$

$$= -\frac{e^{-4}}{4}(\cos 4 + \sin 4) + \frac{e^{6}}{4}(\cos 6 - \sin 6)$$

$$\approx 125.028$$

9. $y = \int x^2 \ln x\, dx$

$dv = x^2\, dx$

$v = \frac{1}{3}x^3$

$u = \ln x$

$du = \frac{1}{x}\, dx$

$$y = (\ln x)\left(\frac{1}{3}x^3\right) - \int\left(\frac{1}{3}x^3\right)\left(\frac{1}{x}\, dx\right)$$

$$y = \frac{1}{3}x^3\ln x - \frac{1}{3}\int x^2\, dx$$

$$y = \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$$

10. $y = \int \frac{1}{\sin\theta} \times \frac{\cos\theta}{\sin\theta} d\theta = \int \frac{\cos\theta d\theta}{\sin^2\theta}$

$u = \sin\theta; du = \cos\theta d\theta$

$\int u^{-2} du = -\frac{1}{u} + C = \frac{-1}{\sin\theta} + C$

.11 (أ) $dv = e^x d\,x$

$v = e^x$

$u = x$

$du = dx$

$$\int xe^x\, dx = xe^x - \int e^x\, dx = xe^x - e^x + C = (x-1)e^x + C$$

(ب) مستخدمًا نتيجة الجزء (أ)

$dv = e^x\, dx$

$v = e^x$

$u = x^2$

$du = 2x\, dx$

$$\int xe^x\, dx = x^2e^x - \int 2xe^x\, dx = x^2e^x - 2(x-1)e^x + C = (x^2 - 2x + 2)e^x + C$$

(جـ) مستخدمًا نتيجة الجزء (ب)

$dv = e^x\, dx$

$v = e^x$

$u = x^3$

$du = 3x^2\, dx$

$$\int x^3 e^x\, dx = x^3 e^x - \int 3x^2 e^x\, dx = x^3 e^x - 3(x^2 - 2x + 2)e^x + C = (x^3 - 3x^2 + 6x - 6)e^x + C$$

(د) $\left[x^n - \frac{d}{dx}x^n + \frac{d^2}{dx^2}x^n - \frac{d^3}{dx^3}x^n + \ldots\ldots\ldots + (-1)^n \frac{d^n}{dx^n}x^n\right]e^x + C$

(هـ) استخدم الاستقراء الرياضي أو البرهان معتمدًا على التكامل الجدولي بالتناوب، بيِّن أن مشتقة إجابة الجزء (د) هي $x^n e^x$

$$\frac{d}{dx}\left[\left(x^n - nx^{n-1} + n(n-1)x^{n-2} - \ldots + (-1)^{n-1}(n!)x + (-1)^n n!\right)e^x + C\right]$$

$$= \left[x^n - nx^{n-1} + n(n-1)x^{n-2} - \ldots + (-1)^{n-1}(n!)x + (-1)^n n!\right]e^x +$$
$$e^x \frac{d}{dx}[x^n - nx^{n-1} + n(n-1)x^{n-2} - \ldots + (-1)^{n-1}(n!)x + (-1)^n n!]$$

$$= [x^n - nx^{n-1} + n(n-1)x^{n-2} - .. + (-1)^{n-1}(n!)x + (-1)^n n!]e^x$$
$$+ [nx^{n-1} - n(n-1)x^{n-2} + n(n-1)(n-2)x^{n-3} -$$
$$... + (-1)^{n-1} n!]e^x = x^n e^x$$

12. لتكن $w = \sqrt{3x+9}$. فإن $dw = \frac{1}{2\sqrt{3x+9}}(3)\, dx$

إذًا $dx = \frac{2}{3}\sqrt{3x+9}\, dw = \frac{2}{3} w\, dw$

$$\int e^{\sqrt{3x+9}}\, dx = \int (e^w)\left(\frac{2}{3} w\, dw\right) = \frac{2}{3}\int w\, e^w\, dw$$

$$dv = e^w\, dw$$
$$v = e^w$$
$$u = w$$
$$du = dw$$

$$\int w\, e^w\, dw = w\, e^w - \int e^w\, dw = w\, e^w - e^w = (w-1)e^w$$

$$\int e^{\sqrt{3x+9}}\, dx = \frac{2}{3}\int w\, e^w\, dw = \frac{2}{3}(w-1)\, e^w = \frac{2}{3}\left(\sqrt{3x+9} - 1\right) e^{\sqrt{3x+9}} + C$$