AERO/MECH 201 Engineering Dynamics Lecture 2

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CHAPTER 11

Kinematics of Particles

The motion of the space shuttle can be described in terms of its *position, velocity,* and *acceleration.* When landing, the pilot of the shuttle needs to consider the wind velocity and the *relative motion* of the shuttle with respect to the wind. The study of motion is known as *kinematics* and is the subject of this chapter.

Problem Set #1

- Problem set #1 covering Chapter 11 will be uploaded early next week.

11.1 INTRODUCTION TO DYNAMICS

Dynamics includes:

1. *Kinematics*, which is the study of the geometry of motion. Kinematics is used to relate displacement, velocity, acceleration, and time, without reference to the cause of the motion.
2. *Kinetics*, which is the study of the relation existing between the forces acting on a body, the mass of the body, and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

RECTILINEAR MOTION OF PARTICLES

11.2 POSITION, VELOCITY, AND ACCELERATION

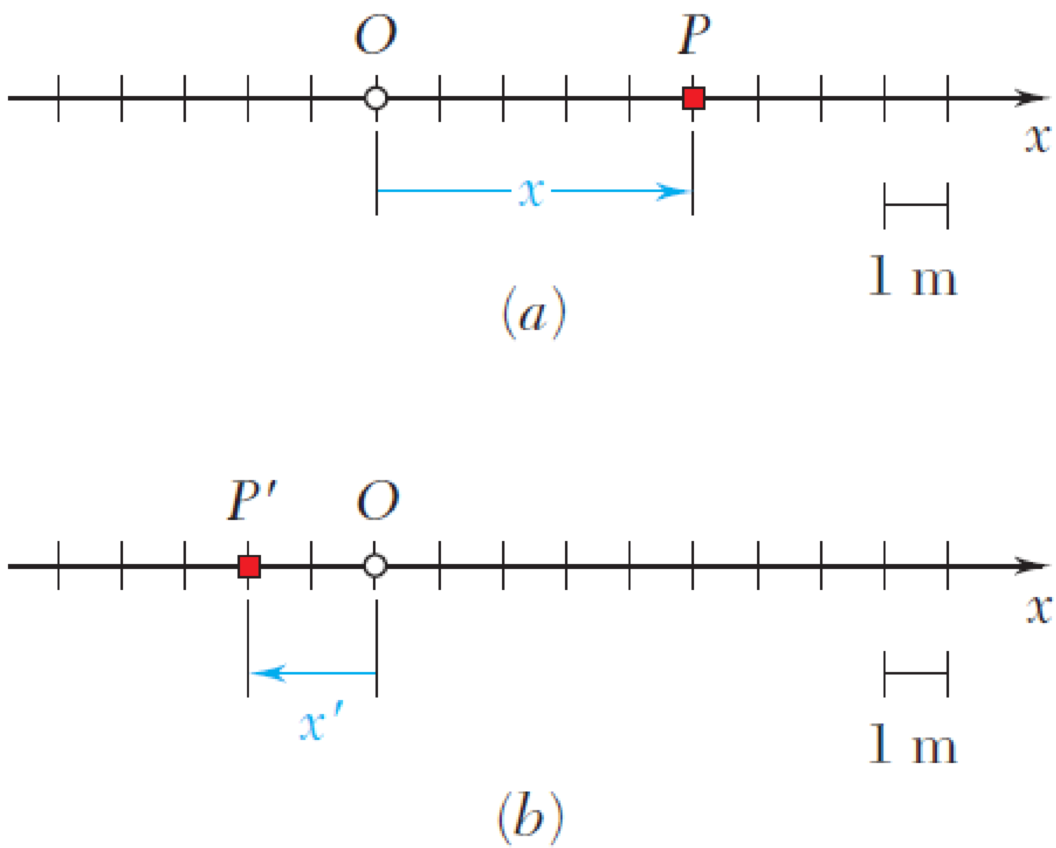


Fig. 11.1

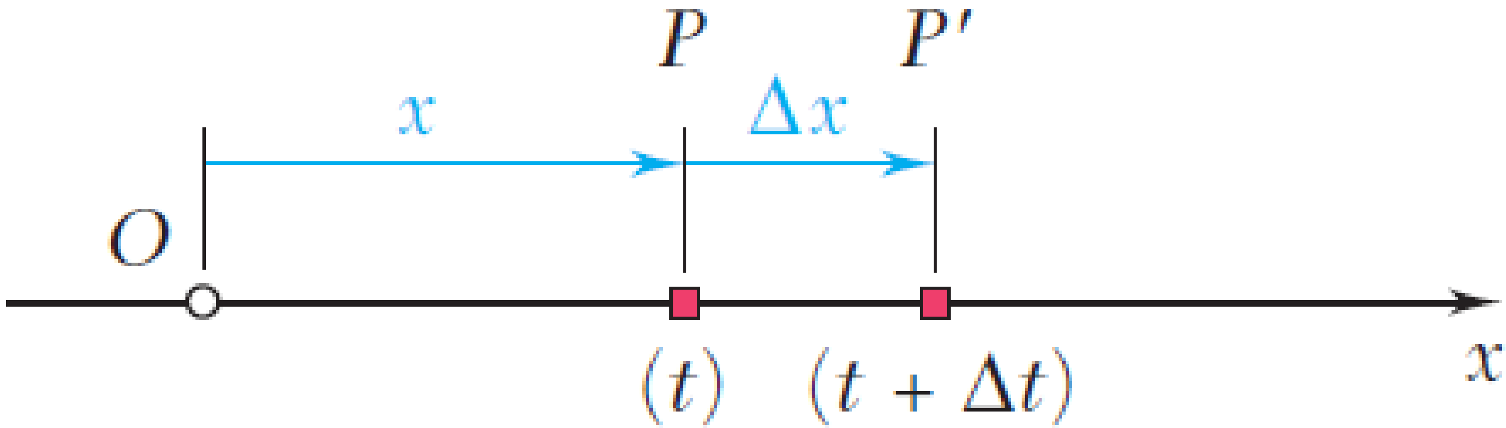


Fig. 11.2

$$\text{Average velocity} = \frac{\Delta x}{\Delta t}$$

$$\text{Instantaneous velocity} = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$\boxed{v = \frac{dx}{dt}}$$

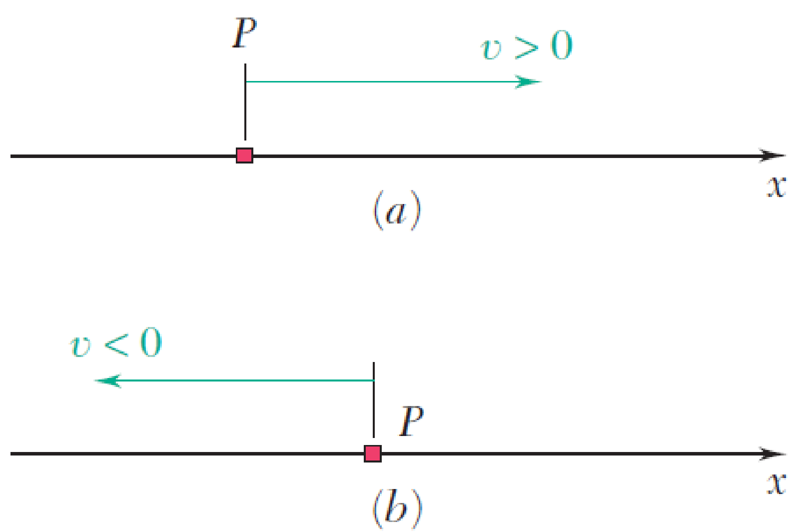


Fig. 11.3

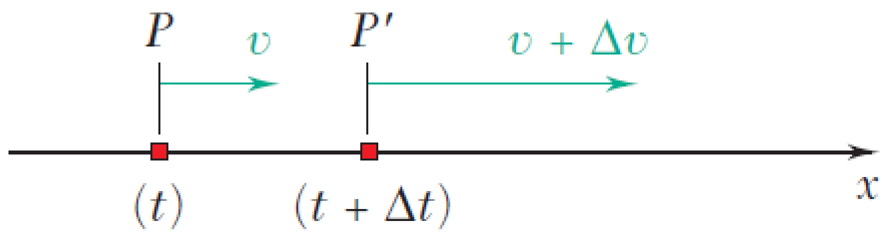


Fig. 11.4

$$\text{Average acceleration} = \frac{\Delta v}{\Delta t}$$

$$\text{Instantaneous acceleration} = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$\boxed{a = \frac{dv}{dt}}$$

$$\boxed{a = \frac{d^2x}{dt^2}}$$

$$\boxed{a = v\frac{dv}{dx}}$$

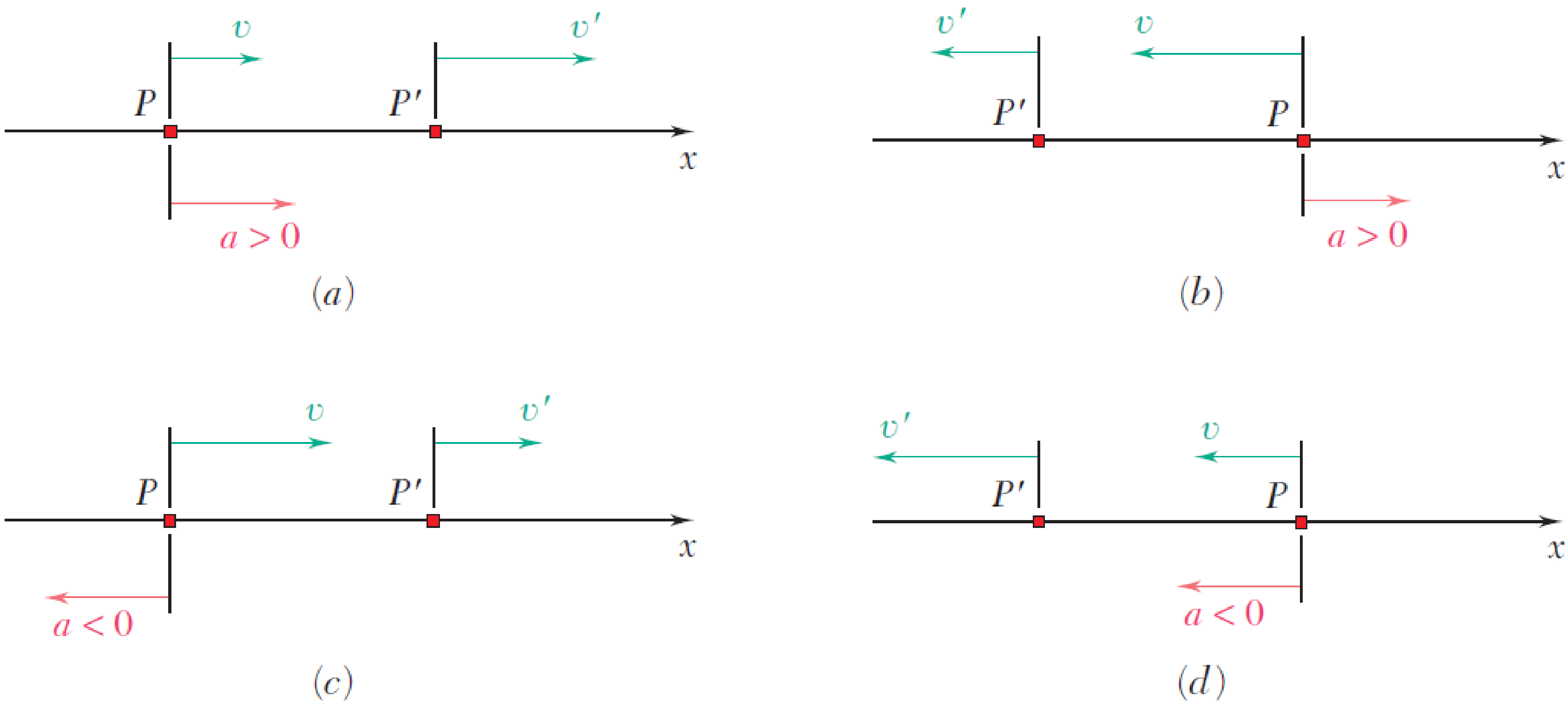


Fig. 11.5

EXAMPLE Consider a particle moving in a straight line, and assume that its position is defined by the equation

$$x = 6t^2 - t^3$$

$$v = \frac{dx}{dt} = 12t - 3t^2$$

$$a = \frac{dv}{dt} = 12 - 6t$$

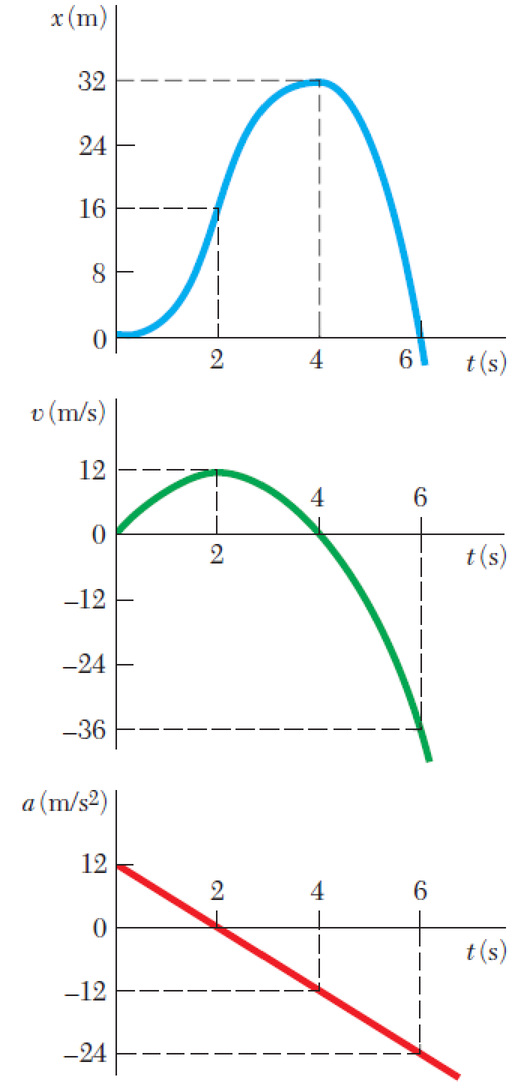


Fig. 11.6

11.3 DETERMINATION OF THE MOTION OF A PARTICLE

SAMPLE PROBLEM 11.1

The position of a particle which moves along a straight line is defined by the relation $x = t^3 - 6t^2 - 15t + 40$, where x is expressed in feet and t in seconds. Determine (*a*) the time at which the velocity will be zero, (*b*) the position and distance traveled by the particle at that time, (*c*) the acceleration of the particle at that time, (*d*) the distance traveled by the particle from $t = 4$ s to $t = 6$ s.

$$x = t^3 - 6t^2 - 15t + 40$$
$$v = \frac{dx}{dt} = 3t^2 - 12t - 15$$
$$a = \frac{dv}{dt} = 6t - 12$$

a. Time at Which $v = 0$. We set $v = 0$ in (2):

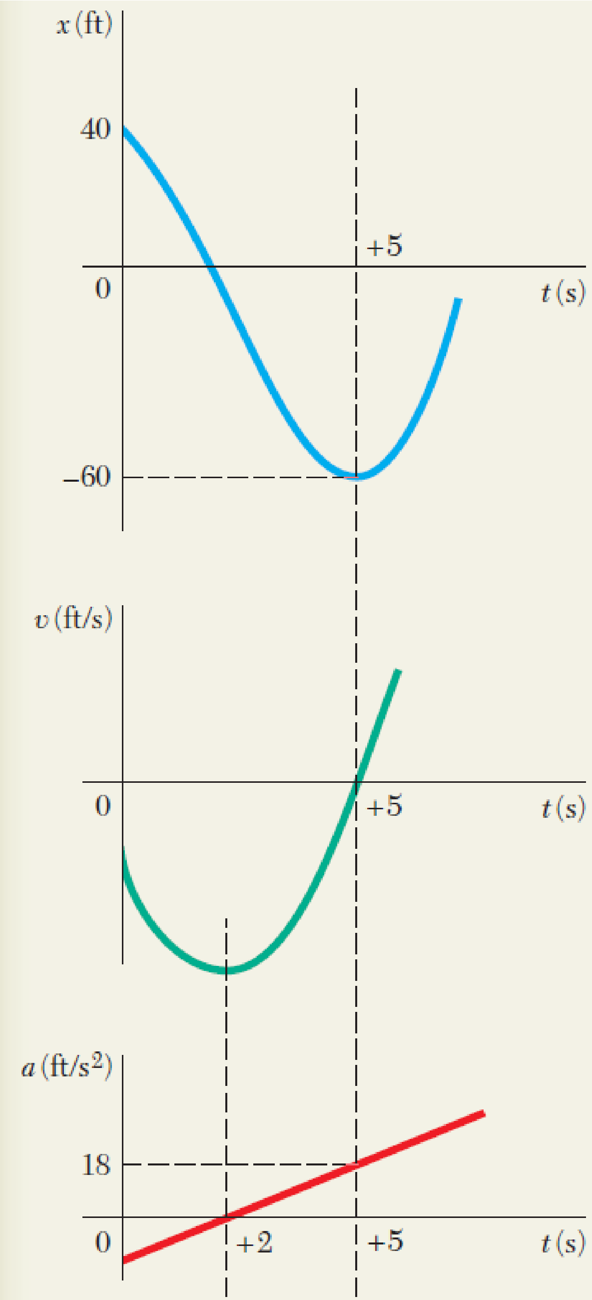
$$3t^2 - 12t - 15 = 0 \qquad t = -1 \text{ s} \qquad \text{and} \qquad t = +5 \text{ s}$$

b. Position and Distance Traveled When $v = 0$. Carrying $t = +5$ s into (1), we have

$$x_5 = (5)^3 - 6(5)^2 - 15(5) + 40 \qquad x_5 = -60 \text{ ft} \quad \blacktriangleleft$$

c. Acceleration When $v = 0$. We substitute $t = +5$ s into (3):

$$a_5 = 6(5) - 12 \qquad a_5 = +18 \text{ ft/s}^2 \quad \blacktriangleleft$$



d. Distance Traveled from $t = 4$ s to $t = 6$ s. The particle moves in the negative direction from $t = 4$ s to $t = 5$ s and in the positive direction from $t = 5$ s to $t = 6$ s; therefore, the distance traveled during each of these time intervals will be computed separately.

From $t = 4$ s to $t = 5$ s: $\quad x_5 = -60$ ft

$$x_4 = (4)^3 - 6(4)^2 - 15(4) + 40 = -52 \text{ ft}$$

$$\begin{aligned}\text{Distance traveled} &= x_5 - x_4 = -60 \text{ ft} - (-52 \text{ ft}) = -8 \text{ ft}\\ &= 8 \text{ ft in the negative direction}\end{aligned}$$

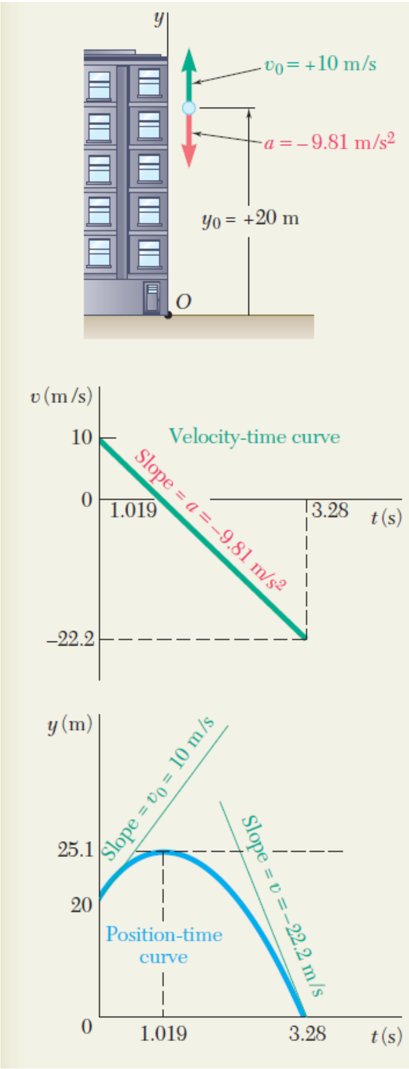
From $t = 5$ s to $t = 6$ s: $\quad x_5 = -60$ ft

$$x_6 = (6)^3 - 6(6)^2 - 15(6) + 40 = -50 \text{ ft}$$

$$\begin{aligned}\text{Distance traveled} &= x_6 - x_5 = -50 \text{ ft} - (-60 \text{ ft}) = +10 \text{ ft}\\ &= 10 \text{ ft in the positive direction}\end{aligned}$$

Total distance traveled from $t = 4$ s to $t = 6$ s is 8 ft + 10 ft $= 18$ ft ◀

Read page 610 and 611



SAMPLE PROBLEM 11.1

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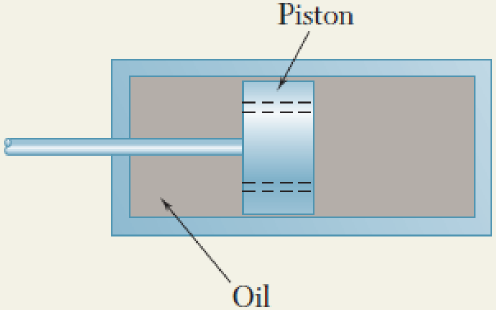
SAMPLE PROBLEM 11.3



The brake mechanism used to reduce recoil in certain types of guns consists essentially of a piston attached to the barrel and moving in a fixed cylinder filled with oil. As the barrel recoils with an initial velocity v_0, the piston moves and oil is forced through orifices in the piston, causing the piston and the barrel to decelerate at a rate proportional to their velocity; that is, $a = -kv$. Express (*a*) v in terms of t, (*b*) x in terms of t, (*c*) v in terms of x. Draw the corresponding motion curves.

11.4 UNIFORM RECTILINEAR MOTION

- Acceleration is ZERO

$$x = x_0 + vt$$

11.5 UNIFORMLY ACCELERATED RECTILINEAR MOTION

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

11.6 MOTION OF SEVERAL PARTICLES

Relative Motion of Two Particles

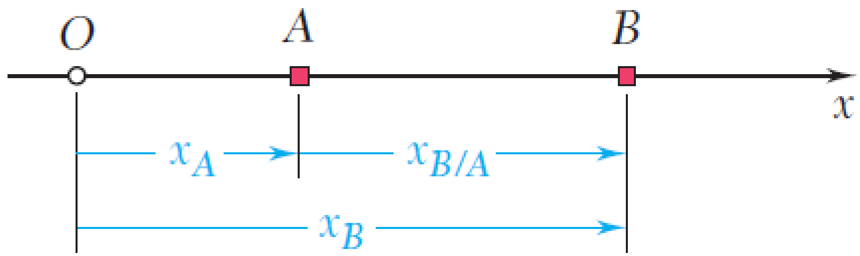


Fig. 11.7

$$x_B = x_A + x_{B/A}$$

$$v_B = v_A + v_{B/A}$$

$$a_B = a_A + a_{B/A}$$

Dependent Motions

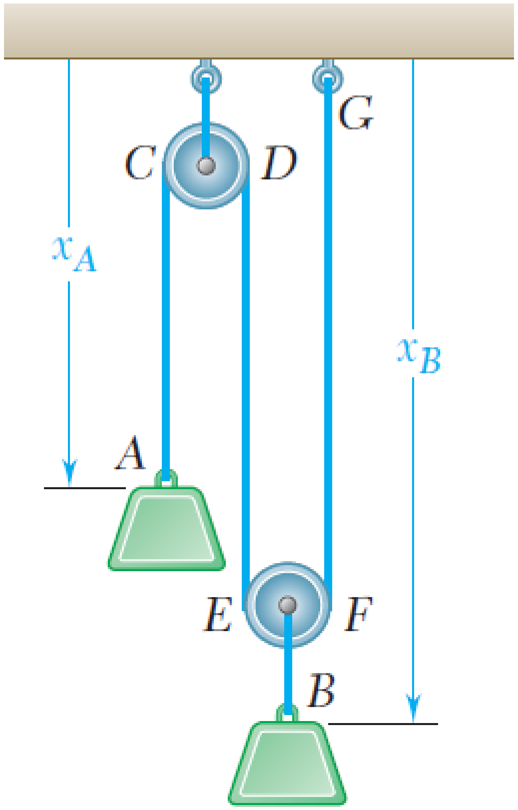
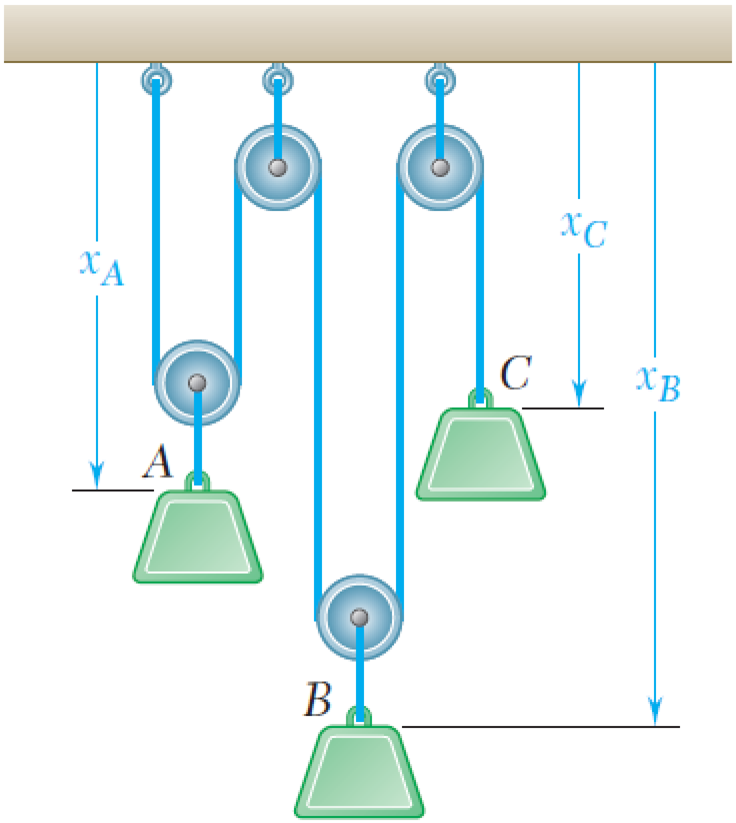


Fig. 11.8

$$x_A + 2x_B = \text{constant}$$

$$\Delta x_B = -\tfrac{1}{2}\Delta x_A$$



$$2x_A + 2x_B + x_C = \text{constant}$$

Fig. 11.9

$$2\frac{dx_A}{dt} + 2\frac{dx_B}{dt} + \frac{dx_C}{dt} = 0 \qquad \text{or} \qquad 2v_A + 2v_B + v_C = 0$$

$$2\frac{dv_A}{dt} + 2\frac{dv_B}{dt} + \frac{dv_C}{dt} = 0 \qquad \text{or} \qquad 2a_A + 2a_B + a_C = 0$$