AERO/MECH 201 Engineering Dynamics Lecture 7

Feb 17th 2013

Dr. Young-Ji Byon

KUSTAR

11.13 TANGENTIAL AND NORMAL COMPONENTS

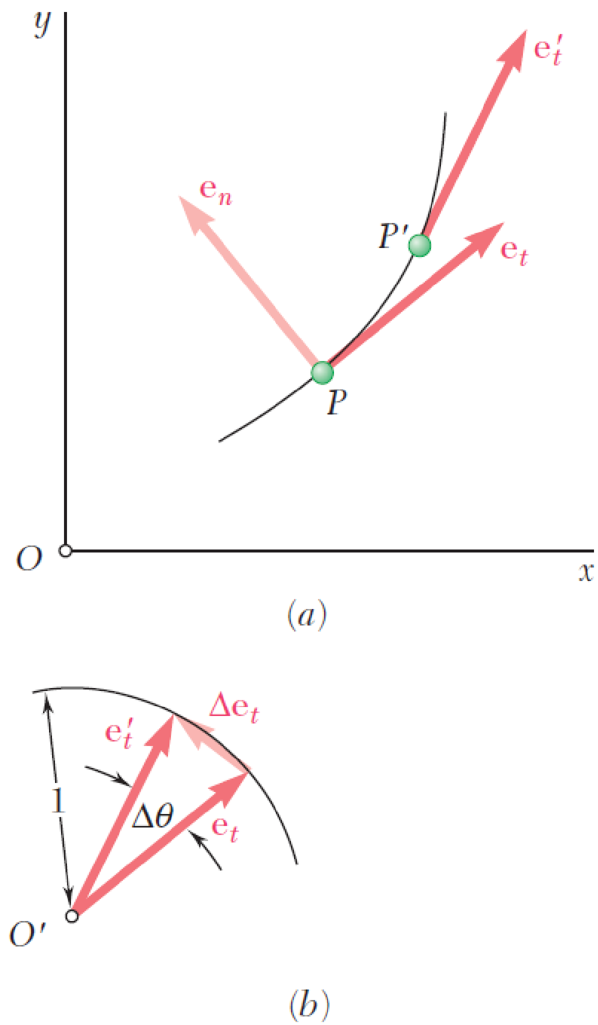
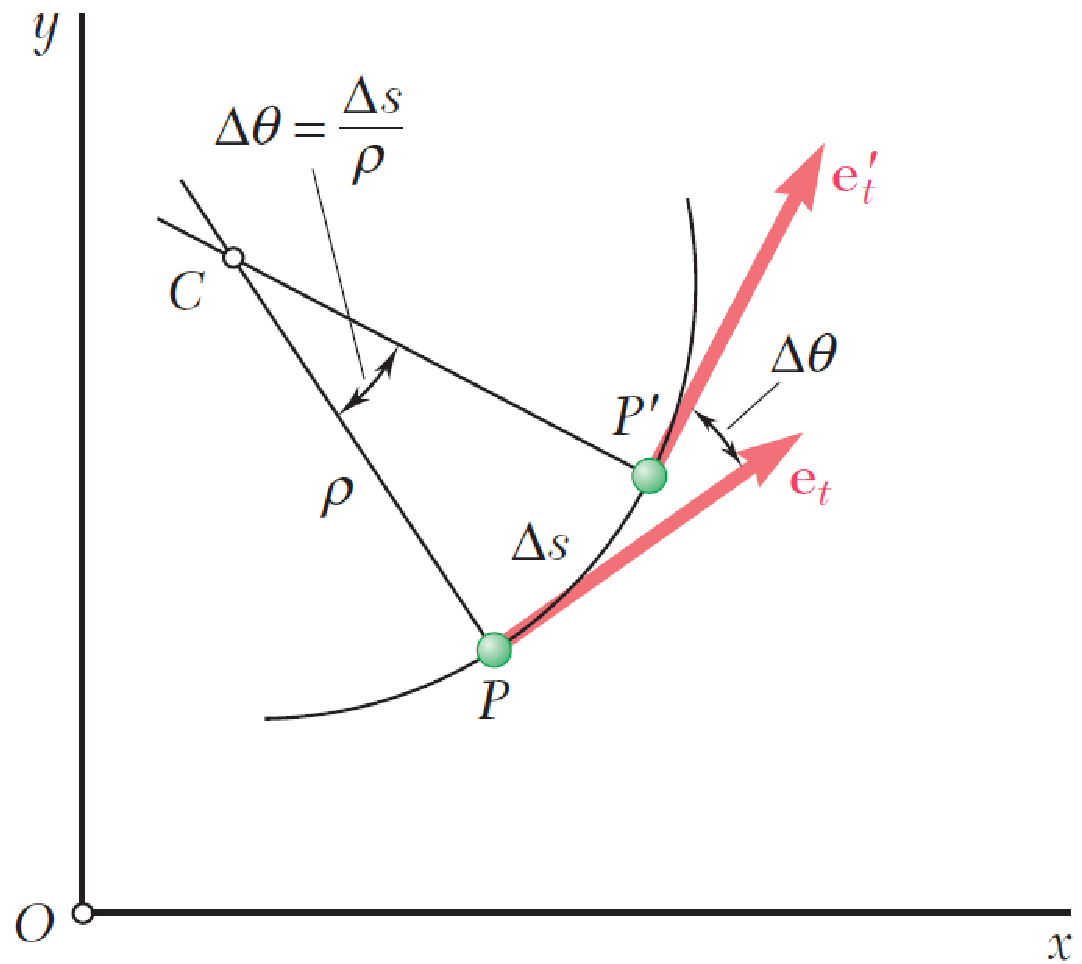


Fig. 11.21

Plane Motion of a Particle

$$\mathbf{e}_n = \frac{d\mathbf{e}_t}{d\theta}$$



y
$\Delta\theta = \frac{\Delta s}{\rho}$
$\mathbf{e}'_t$
C
$\Delta\theta$
P'
$\mathbf{e}_t$
ρ
Δs
P
O
x

Fig. 11.22

$$\mathbf{v} = v\mathbf{e}_t$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{dv}{dt}\mathbf{e}_t + v\frac{d\mathbf{e}_t}{dt}$$

$$\frac{d\mathbf{e}_t}{dt} = \frac{d\mathbf{e}_t}{d\theta}\frac{d\theta}{ds}\frac{ds}{dt}$$

$d\theta/ds$ is equal to $1/\rho$

$$\frac{d\mathbf{e}_t}{dt} = \frac{v}{\rho}\mathbf{e}_n$$

$$\mathbf{a} = \frac{dv}{dt}\mathbf{e}_t + \frac{v^2}{\rho}\mathbf{e}_n$$

$$a_t = \frac{dv}{dt} \qquad a_n = \frac{v^2}{\rho}$$

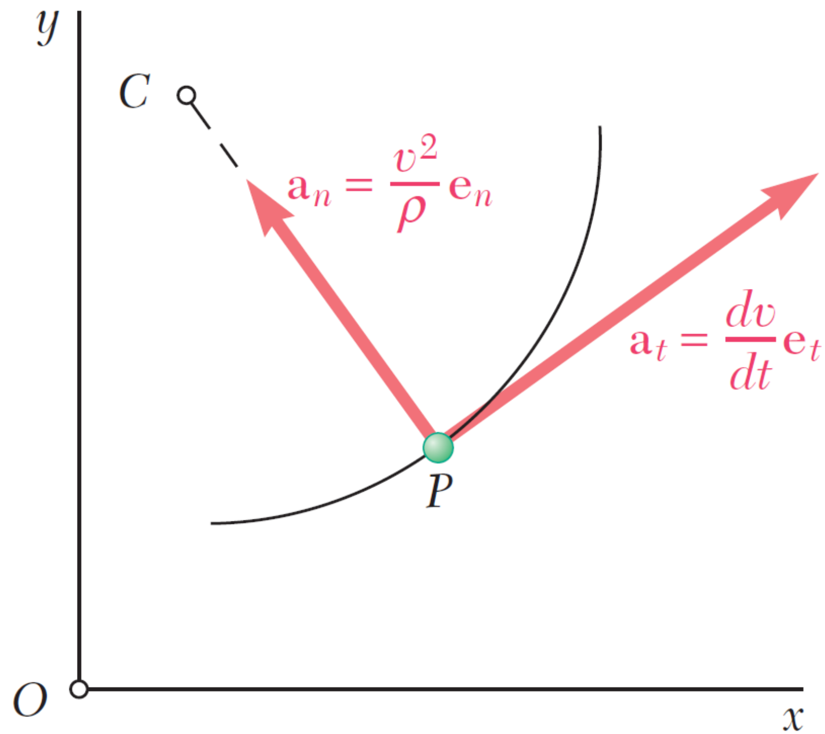


Fig. 11.23

Motion of a Particle in Space

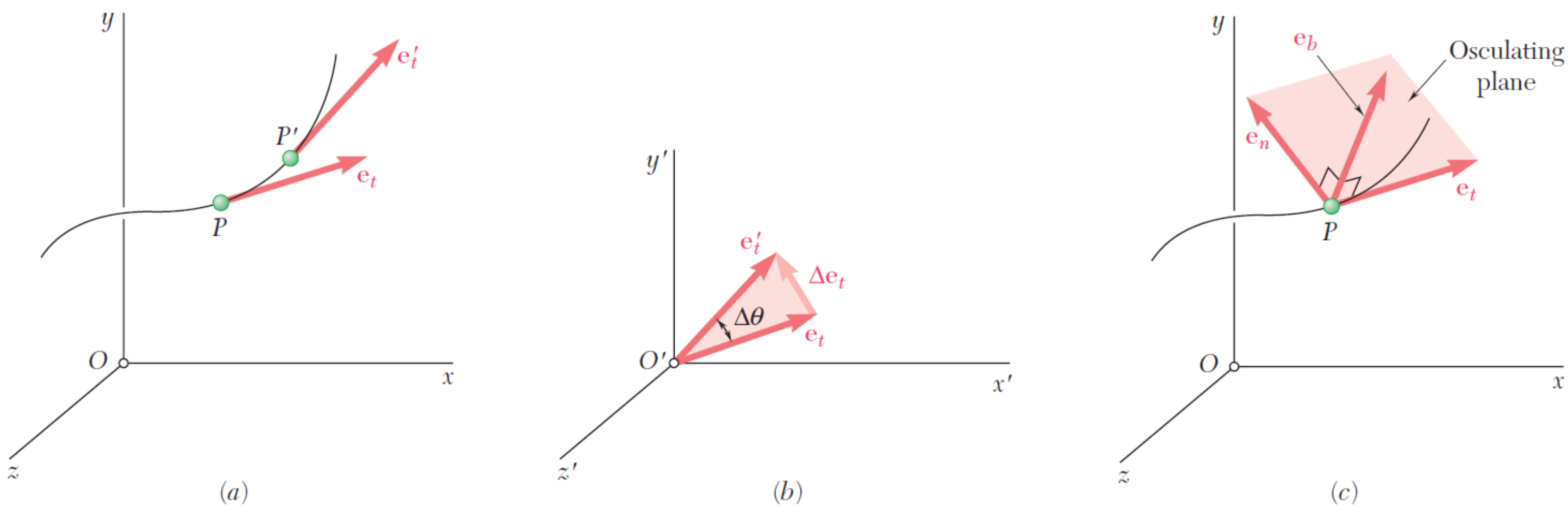


Fig. 11.24

11.14 RADIAL AND TRANSVERSE COMPONENTS

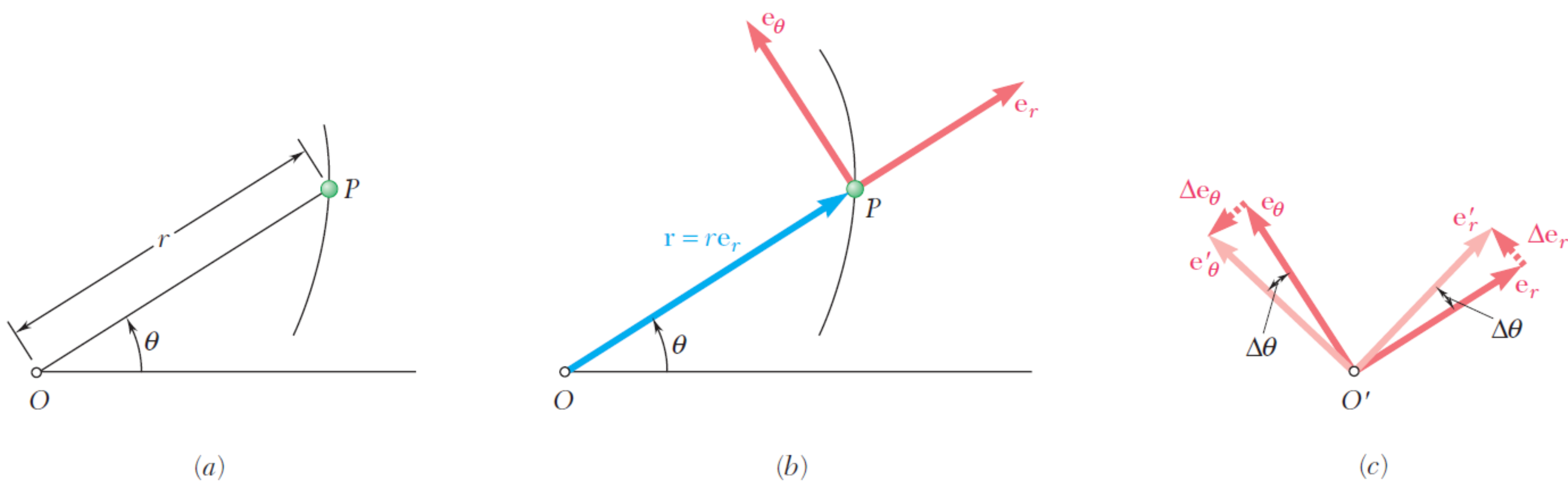
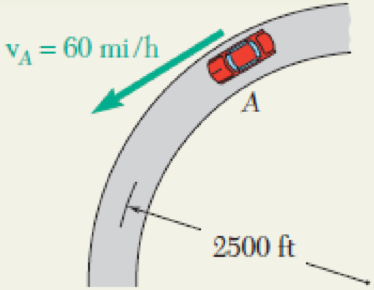


Fig. 11.25

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$

$$\boldsymbol{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$



SAMPLE PROBLEM 11.10

A motorist is traveling on a curved section of highway of radius 2500 ft at the speed of 60 mi/h. The motorist suddenly applies the brakes, causing the automobile to slow down at a constant rate. Knowing that after 8 s the speed has been reduced to 45 mi/h, determine the acceleration of the automobile immediately after the brakes have been applied.

SOLUTION

Tangential Component of Acceleration. First the speeds are expressed in ft/s.

$$60 \text{ mi/h} = \left(60 \frac{\text{mi}}{\text{h}}\right)\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 88 \text{ ft/s}$$

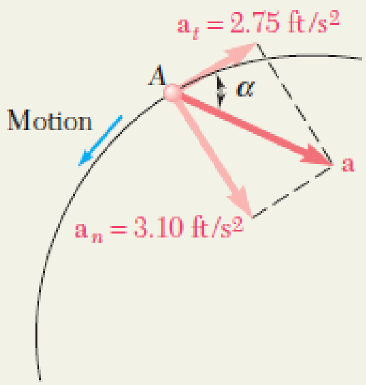
$$45 \text{ mi/h} = 66 \text{ ft/s}$$

Since the automobile slows down at a constant rate, we have

$$a_t = \text{average } a_t = \frac{\Delta v}{\Delta t} = \frac{66 \text{ ft/s} - 88 \text{ ft/s}}{8 \text{ s}} = -2.75 \text{ ft/s}^2$$

Normal Component of Acceleration. Immediately after the brakes have been applied, the speed is still 88 ft/s, and we have

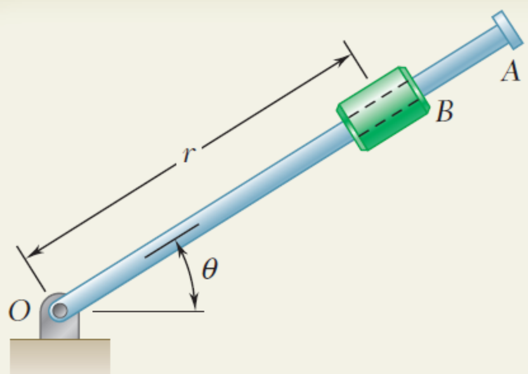
$$a_n = \frac{v^2}{\rho} = \frac{(88 \text{ ft/s})^2}{2500 \text{ ft}} = 3.10 \text{ ft/s}^2$$



Magnitude and Direction of Acceleration. The magnitude and direction of the resultant $\mathbf{a}$ of the components $\mathbf{a}_n$ and $\mathbf{a}_t$ are

$$\tan \alpha = \frac{a_n}{a_t} = \frac{3.10 \text{ ft/s}^2}{2.75 \text{ ft/s}^2} \qquad \alpha = 48.4° \blacktriangleleft$$

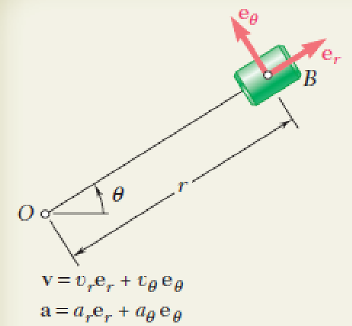
$$a = \frac{a_n}{\sin \alpha} = \frac{3.10 \text{ ft/s}^2}{\sin 48.4°} \qquad \mathbf{a} = 4.14 \text{ ft/s}^2 \blacktriangleleft$$



SAMPLE PROBLEM 11.12

The rotation of the 0.9-m arm OA about O is defined by the relation $\theta = 0.15t^2$, where θ is expressed in radians and t in seconds. Collar B slides along the arm in such a way that its distance from O is $r = 0.9 - 0.12t^2$, where r is expressed in meters and t in seconds. After the arm OA has rotated through 30°, determine (*a*) the total velocity of the collar, (*b*) the total acceleration of the collar, (*c*) the relative acceleration of the collar with respect to the arm.

SOLUTION

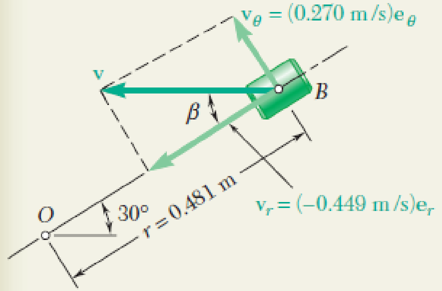


Time t at which $\theta = 30°$. Substituting $\theta = 30° = 0.524$ rad into the expression for θ, we obtain

$$\theta = 0.15t^2 \qquad 0.524 = 0.15t^2 \qquad t = 1.869 \text{ s}$$

Equations of Motion. Substituting $t = 1.869$ s in the expressions for r, θ, and their first and second derivatives, we have

$$\begin{aligned} r &= 0.9 - 0.12t^2 = 0.481 \text{ m} & \theta &= 0.15t^2 = 0.524 \text{ rad} \\ \dot{r} &= -0.24t = -0.449 \text{ m/s} & \dot{\theta} &= 0.30t = 0.561 \text{ rad/s} \\ \ddot{r} &= -0.24 = -0.240 \text{ m/s}^2 & \ddot{\theta} &= 0.30 = 0.300 \text{ rad/s}^2 \end{aligned}$$

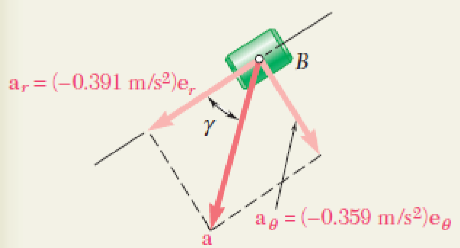


a. Velocity of B. Using Eqs. (11.45), we obtain the values of v_r and v_θ when $t = 1.869$ s.

$$\begin{aligned} v_r &= \dot{r} = -0.449 \text{ m/s} \\ v_\theta &= r\dot{\theta} = 0.481(0.561) = 0.270 \text{ m/s} \end{aligned}$$

Solving the right triangle shown, we obtain the magnitude and direction of the velocity,

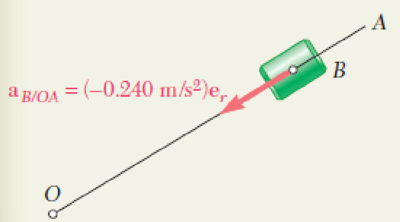
$$v = 0.524 \text{ m/s} \qquad \beta = 31.0° \quad ◀$$



b. Acceleration of B. Using Eqs. (11.46), we obtain

$$\begin{aligned} a_r &= \ddot{r} - r\dot{\theta}^2 \\ &= -0.240 - 0.481(0.561)^2 = -0.391 \text{ m/s}^2 \\ a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} \\ &= 0.481(0.300) + 2(-0.449)(0.561) = -0.359 \text{ m/s}^2 \end{aligned}$$

$$a = 0.531 \text{ m/s}^2 \qquad \gamma = 42.6° \quad ◀$$



c. Acceleration of B with Respect to Arm OA. We note that the motion of the collar with respect to the arm is rectilinear and defined by the coordinate r. We write

$$a_{B/OA} = \ddot{r} = -0.240 \text{ m/s}^2$$

$$a_{B/OA} = 0.240 \text{ m/s}^2 \text{ toward } O. \quad ◀$$