AERO/MECH 201
Engineering Dynamics Lecture 8

Feb 21st 2013

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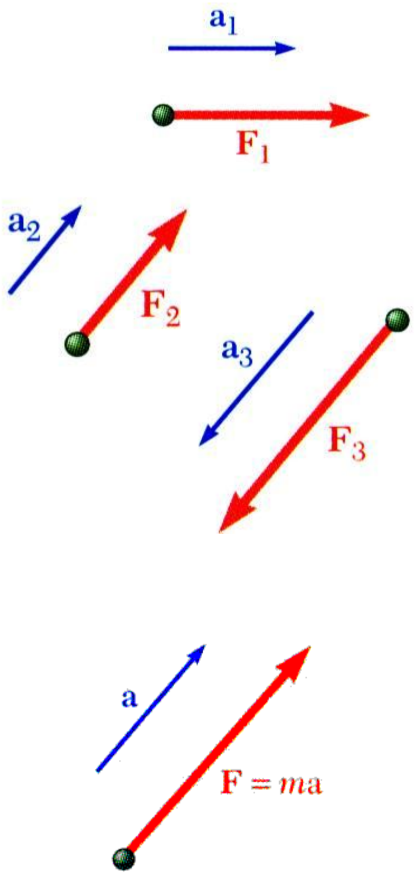
Newton's Second Law of Motion



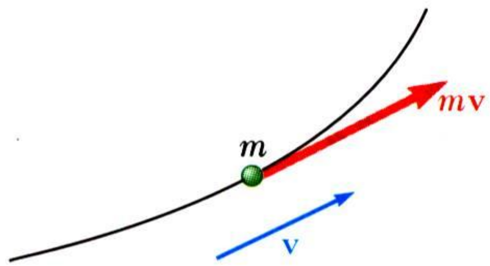
- *Newton's Second Law*: If the *resultant force* acting on a particle is not zero, the particle will have an acceleration *proportional to the magnitude of resultant* and in the *direction of the resultant.*
- Consider a particle subjected to constant forces,

$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \frac{F_3}{a_3} = \cdots = \text{constant} = \text{mass}, m$$

- When a particle of mass m is acted upon by a force $\vec{F}$, the acceleration of the particle must satisfy

$$\vec{F} = m\vec{a}$$

Linear Momentum of a Particle



- Replacing the acceleration by the derivative of the velocity yields

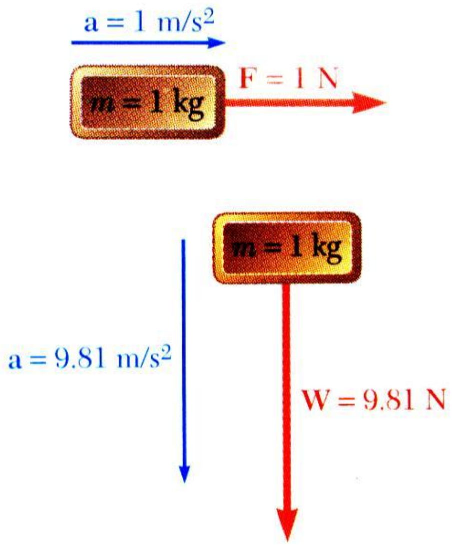
$$\sum \vec{F} = m\frac{d\vec{v}}{dt}$$

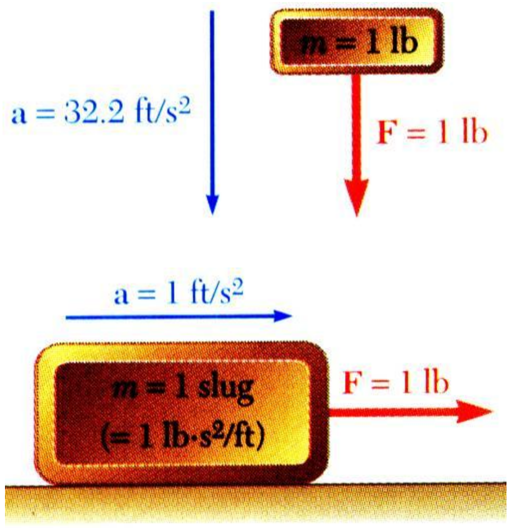
$$= \frac{d}{dt}(m\vec{v}) = \frac{d\vec{L}}{dt}$$

$$\vec{L} = \text{linear momentum of the particle}$$

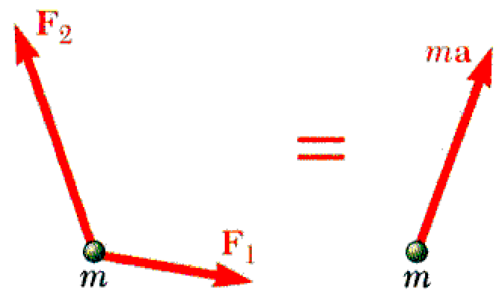
- *Linear Momentum Conservation Principle*:
  If the resultant force on a particle is zero, the linear momentum of the particle remains constant in both magnitude and direction.

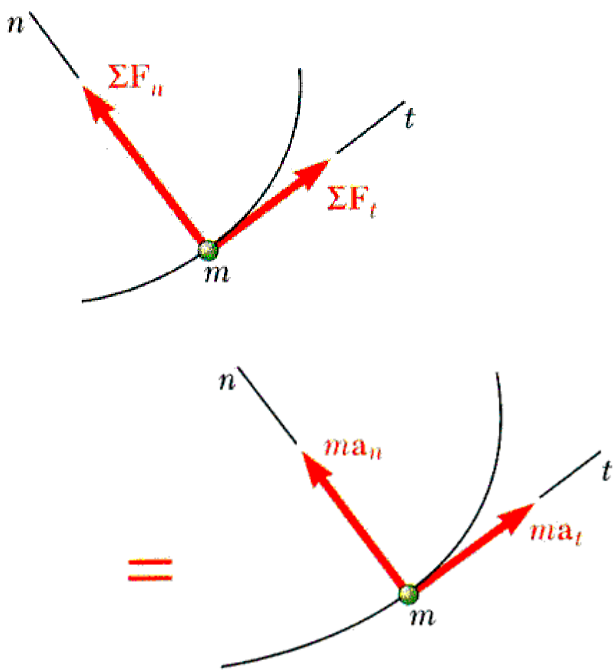
Systems of Units





Equations of Motion





- Newton’s second law provides

$$\sum \vec{F} = m\vec{a}$$

- Solution for particle motion is facilitated by resolving vector equation into scalar component equations, e.g., for rectangular components,

$$\sum\left(F_x\vec{i} + F_y\vec{j} + F_z\vec{k}\right) = m\left(a_x\vec{i} + a_y\vec{j} + a_z\vec{k}\right)$$

$$\sum F_x = ma_x \quad \sum F_y = ma_y \quad \sum F_z = ma_z$$

$$\sum F_x = m\ddot{x} \quad \sum F_y = m\ddot{y} \quad \sum F_z = m\ddot{z}$$

- For tangential and normal components,

$$\sum F_t = ma_t \quad \sum F_n = ma_n$$

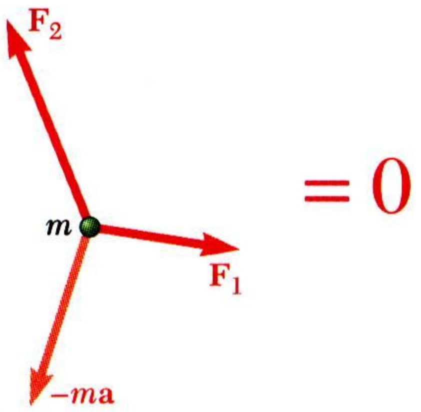
$$\sum F_t = m\frac{dv}{dt} \quad \sum F_n = m\frac{v^2}{\rho}$$

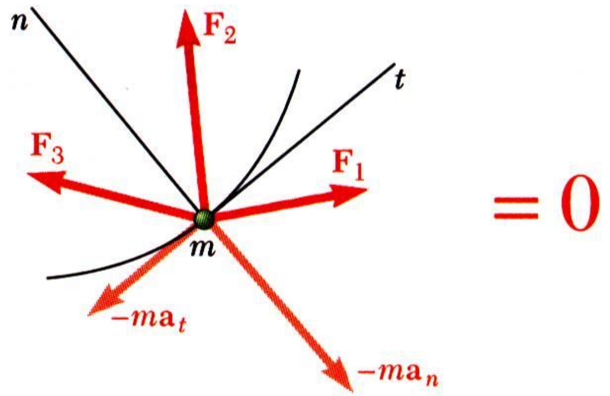
Dynamic Equilibrium

- Alternate expression of Newton's second law,

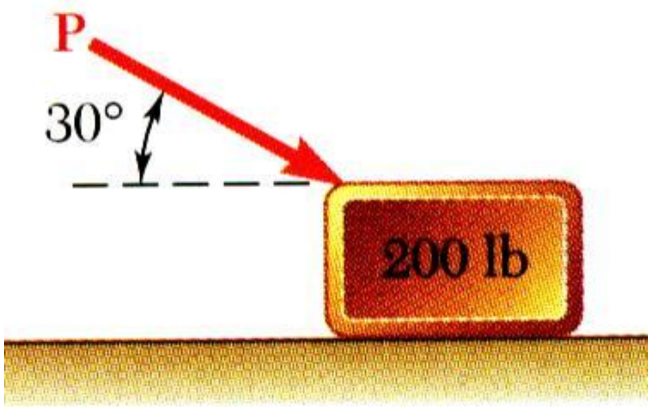
$$\sum \vec{F} - m\vec{a} = 0$$

$$-m\vec{a} \equiv inertial\ vector$$



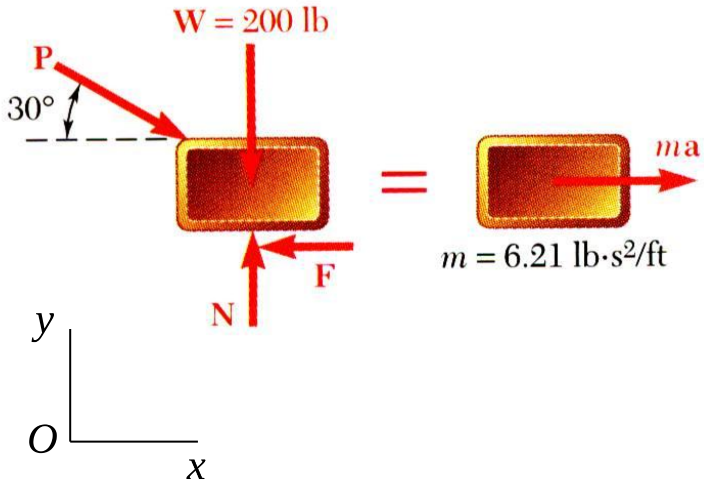


Sample Problem 12.1



A 200-lb block rests on a horizontal plane. Find the magnitude of the force ***P*** required to give the block an acceleration of 10 ft/s^2 to the right. The coefficient of kinetic friction between the block and plane is $\mu_k = 0.25$.

Sample Problem 12.1



$$m = \frac{W}{g} = \frac{200\,\text{lb}}{32.2\,\text{ft}/\text{s}^2}$$

$$= 6.21\,\frac{\text{lb}\cdot\text{s}^2}{\text{ft}}$$

$$F = \mu_k N$$

$$= 0.25N$$

SOLUTION:

- Resolve the equation of motion for the block into two rectangular component equations.

$$\sum F_x = ma:$$

$$P\cos 30° - 0.25N = \left(6.21\,\text{lb}\cdot\text{s}^2/\text{ft}\right)\left(10\,\text{ft}/\text{s}^2\right)$$

$$= 62.1\,\text{lb}$$

$$\sum F_y = 0:$$

$$N - P\sin 30° - 200\,\text{lb} = 0$$

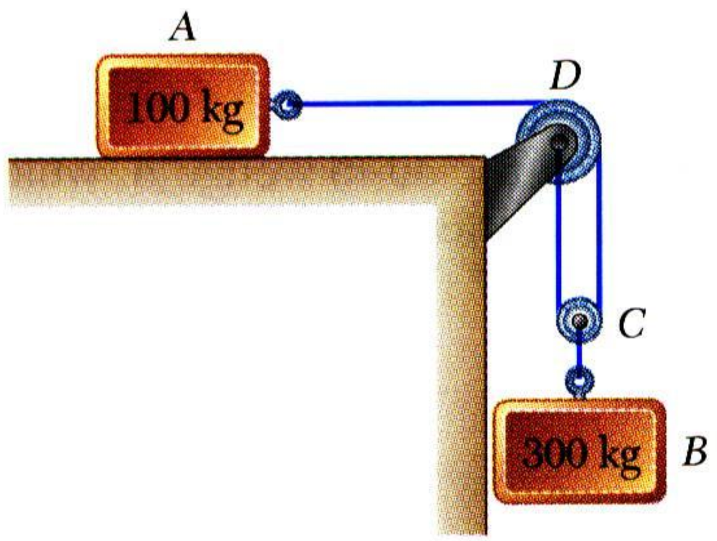
- Unknowns consist of the applied force ***P*** and the normal reaction ***N*** from the plane. The two equations may be solved for these unknowns.

$$N = P\sin 30° + 200\,\text{lb}$$

$$P\cos 30° - 0.25\left(P\sin 30° + 200\,\text{lb}\right) = 62.1\,\text{lb}$$

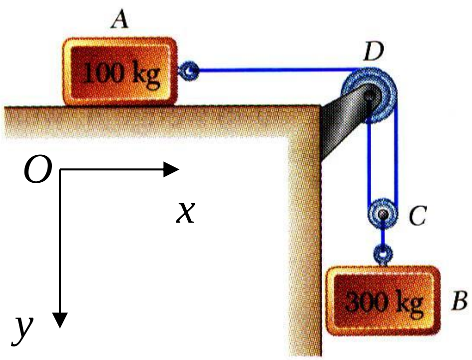
$$\boxed{P = 151\,\text{lb}}$$

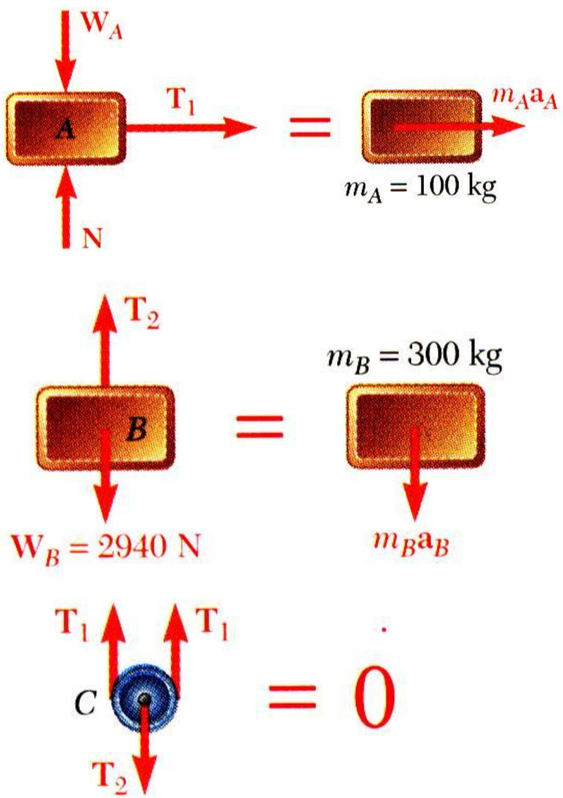
Sample Problem 12.3



The two blocks shown start from rest. The horizontal plane and the pulley are frictionless, and the pulley is assumed to be of negligible mass. Determine the acceleration of each block and the tension in the cord.

Sample Problem 12.3





SOLUTION:

- Write the kinematic relationships for the dependent motions and accelerations of the blocks.

$$y_B = \tfrac{1}{2} x_A \qquad a_B = \tfrac{1}{2} a_A$$

- Write equations of motion for blocks and pulley.

$$\sum F_x = m_A a_A :$$

$$T_1 = (100\,\text{kg})a_A$$

$$\sum F_y = m_B a_B :$$

$$m_B g - T_2 = m_B a_B$$

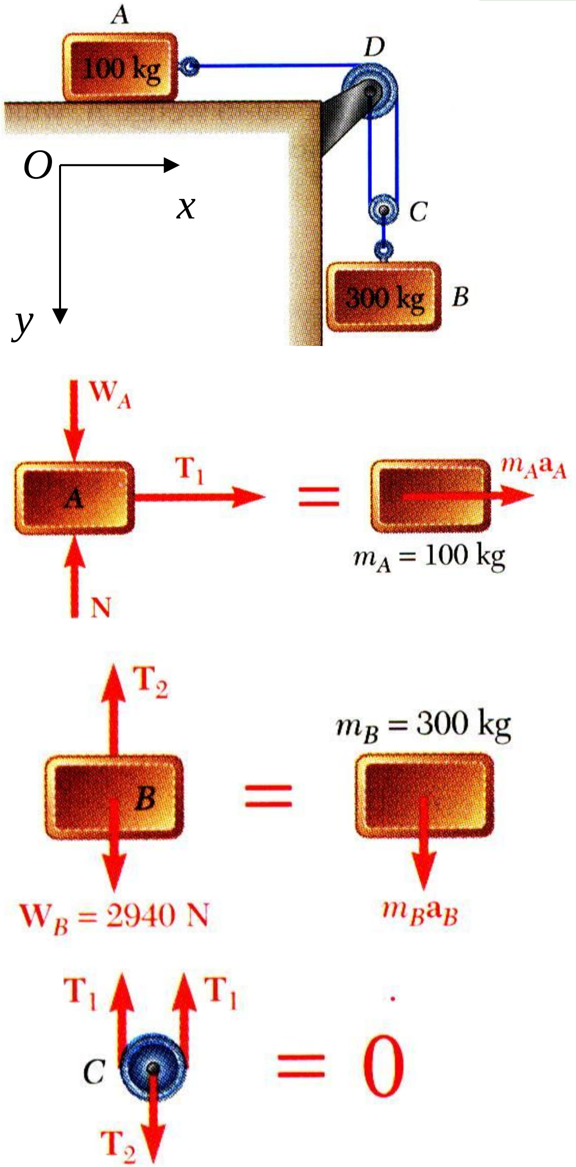
$$(300\,\text{kg})\left(9.81\,\text{m/s}^2\right) - T_2 = (300\,\text{kg})a_B$$

$$T_2 = 2940\,\text{N} - (300\,\text{kg})a_B$$

$$\sum F_y = m_C a_C = 0 :$$

$$T_2 - 2T_1 = 0$$

Sample Problem 12.3



- Combine kinematic relationships with equations of motion to solve for accelerations and cord tension.

$$y_B = \tfrac{1}{2} x_A \qquad a_B = \tfrac{1}{2} a_A$$

$$T_1 = (100\,\text{kg})a_A$$

$$T_2 = 2940\,\text{N} - (300\,\text{kg})a_B$$
$$= 2940\,\text{N} - (300\,\text{kg})\left(\tfrac{1}{2} a_A\right)$$

$$T_2 - 2T_1 = 0$$
$$2940\,\text{N} - (150\,\text{kg})a_A - 2(100\,\text{kg})a_A = 0$$

$$\boxed{\begin{aligned} a_A &= 8.40\,\text{m/s}^2 \\ a_B &= \tfrac{1}{2} a_A = 4.20\,\text{m/s}^2 \\ T_1 &= (100\,\text{kg})a_A = 840\,\text{N} \\ T_2 &= 2T_1 = 1680\,\text{N} \end{aligned}}$$