



Higher Colleges of Technology  
Fujairah Men's College

## MTH 2503 - Fall 2014

### Project

WEIGHTED AT **15 %** OF FINAL COURSE GRADE

#### Academic Honesty Declaration

**Project:** This is entirely my own work except where I have noted other sources in the text and listed them at the assignment's end. I have not submitted this work to HCT before. I understand that I may be interviewed on my submission.

THE PENALTY FOR CHEATING AT HCT IS SEVERE AND INCLUDES PERMANENT DISMISSAL FROM THE COLLEGE. I have read the above information and understand my responsibilities with regard to academic honesty during this project

| Group members | ID |
|---------------|----|
|               |    |
|               |    |
|               |    |
|               |    |

| Standards | Completeness and correctness | Quality | Technology | Creativity | Total |
|-----------|------------------------------|---------|------------|------------|-------|
| Mark      |                              |         |            |            |       |

## 1. Application of Systems of linear equations

Determine the currents  $I_1$ ,  $I_2$  and  $I_3$  for the electrical network shown in Figure 1.12.

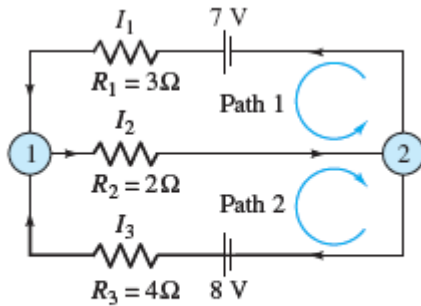


Figure 1.12

## 2. Application of Matrix operations

A cryptogram is a message written according to a secret code (the Greek word kryptos means “hidden”). The following describes a method of using matrix multiplication to encode and decode messages. To begin, assign a number to each letter in the alphabet (with 0 assigned to a blank space), as follows.

|    |   |    |   |
|----|---|----|---|
| 0  | - | 14 | N |
| 1  | A | 15 | O |
| 2  | B | 16 | P |
| 3  | C | 17 | Q |
| 4  | D | 18 | R |
| 5  | E | 19 | S |
| 6  | F | 20 | T |
| 7  | G | 21 | U |
| 8  | H | 22 | V |
| 9  | I | 23 | W |
| 10 | J | 24 | X |
| 11 | K | 25 | Y |
| 12 | L | 26 | Z |
| 13 | M |    |   |

- (a) Write the uncoded row matrices of the indicated size for the given message. Then encode the message using the matrix A

- Message : HELP IS COMING

- Row matrix size :  $1 \times 4$

- Encoding Matrix  $A = \begin{bmatrix} -2 & 3 & -1 & -1 \\ -1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 2 \\ 3 & 1 & -2 & -4 \end{bmatrix}$

- (b) Decoding a Message use  $A^{-1}$  to decode the cryptogram.

$$A = \begin{bmatrix} 3 & -4 & 2 \\ 0 & 2 & 1 \\ 4 & -5 & 3 \end{bmatrix}$$

112   -140   83   19   -25   13   72   -76   61   95   -118   71   20   21   38   35   -23   36  
42   -48   32

### 3. Application of Determinants

- (a) Finding the area of a Triangle with the given vertices(1,1), (−1,1), (0, −2)
- (b) Determine whether the points are collinear (−1, −3), (−4,7), (2, −13)
- (c) Find an equation of the line passing through the given points(1,4), (3,4)
- (d) Find the volume of the Tetrahedron with the given vertices(0,0,0), (0,2,0), (3,0,0), (1,1,4)
- (e) Find an equation of the plane passing through the given points (0, −1,0), (1,1,0), (2,1,2)

### 4. Application of Eigenvalues and Eigenvectors

**Population Growth Model** A population has the following characteristics.

- (a) A total of 60% of the population survives the first year. Of that 60%, 50% survives the second year. The maximum life span is 3 years.
- (b) The average number of offspring for each member of the population is 2 the first year, 5 the second year, and 2 the third year.

The population now consists of 100 members in each of the three age classes. How many members will there be in each age class in 1 year? in 2 years?

### 5. Modeling with First Order Differential Equations

Heat transfer from a body to its surroundings by radiation, based on the Stefan– Boltzmann's law, is described by the differential equation

$$\frac{du}{dt} = -\alpha(u^4 - T^4) \quad (i)$$

where  $u(t)$  is the absolute temperature of the body at time  $t$ ,  $T$  is the absolute temperature of the surroundings, and  $\alpha$  is a constant depending on the physical parameters of the body.

However, if  $u$  is much larger than  $T$ , then solutions of Eq. (i) are well approximated by solutions of the simpler equation

$$\frac{du}{dt} = -\alpha u^4 \quad (ii)$$

Suppose that a body with initial temperature 2000 K is surrounded by a medium with temperature 300 K and that  $\alpha = 2.0 \times 10^{-12} \text{ K}^{-3}/\text{s}$ .

- (a) Determine the temperature of the body at any time by solving Eq. (ii).
- (b) Plot the graph of  $u$  versus  $t$ .
- (c) Find the time  $\tau$  at which  $u(\tau) = 600$  - that is, twice the ambient temperature. Up to this time the error in using Eq.(ii) to approximate the solutions of Eq. (i) is no more than 1%.

### 6. Mechanical and Electrical vibrations with S.O.D.E

A series circuit has a capacitor of  $10^{-5} \text{ F}$ , a resistor of  $3 \times 10^2 \Omega$ , and an inductor of 0.2 H. The initial charge on the capacitor is  $10^{-6} \text{ C}$  and there is no initial current. Find the charge  $Q$  on the capacitor at any time  $t$ .

#### Instructions

- This project is a group project. Each group member is expected to participate effectively.
- Answer all problems in MS word. Use Times New Roman font with font size 10. Use math equation for math symbols and equations.
- Use geogebra for plotting
- Show all of the work
- Submission date is **Nov, 30<sup>th</sup>**
- Marking scheme will be according to the completeness the tasks, quality, correctness, creativity, and using Technology